High School Statistics Curriculum

**Course Description:** A concentration on the analysis of both descriptive and inferential statistics with probability, estimation, averages and variations, distributions, hypothesis testing and correlation emphasized. Students work with activities including probabilities, testing ideas hypothesis, a project over distributions and the accumulation of data. The concepts learned will be used in many college degree programs and career choices.

**Scope and Sequence:**

<table>
<thead>
<tr>
<th>Timeframe</th>
<th>Unit</th>
<th>Instructional Topics</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 weeks</td>
<td>Unit 1: Exploring Data</td>
<td>Topic 1: Explore Data&lt;br&gt;Topic 2: Modeling Distributions of Data&lt;br&gt;Topic 3: Describing Relationships</td>
</tr>
<tr>
<td>2 week</td>
<td>Unit 2: Sampling and Experimentation</td>
<td>Topic 1: Sampling&lt;br&gt;Topic 2: Experimenting</td>
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<tr>
<td>5 weeks</td>
<td>Unit 3: Anticipating Patterns</td>
<td>Topic 1: Probability&lt;br&gt;Topic 2: Random Variables&lt;br&gt;Topic 3: Sampling Distributions</td>
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<tr>
<td>7 weeks</td>
<td>Unit 4: Statistical Inference</td>
<td>Topic 1: Confidence Interval&lt;br&gt;Topic 2: Significance Tests&lt;br&gt;Topic 3: Comparing Two Populations</td>
</tr>
</tbody>
</table>
Unit 1: Exploring Data

Subject: Statistics
Grade: 11, 12
Name of Unit: Exploring Data
Length of Unit: 4 weeks
Overview of Unit: Exploratory analysis of data makes use of graphical and numerical techniques to study patterns and departures from patterns. Emphasis should be placed on interpreting information from graphical and numerical displays and summaries.

Priority Standards for unit:

● APSTATS.I.A: Constructing and interpreting graphical displays of distributions of univariate data (dotplot, stemplot, histogram, cumulative frequency plot)
  1. Center and spread
  2. Clusters and gaps
  3. Outliers and other unusual features
  4. Shape
● APSTATS.I.B: Summarizing distributions of univariate data
  1. Measuring center: median, mean
  2. Measuring spread: range, interquartile range, standard deviation
  3. Measuring position: quartiles, percentiles, standardized scores (z-scores)
  4. Using boxplots
  5. The effect of changing units on summary measures
● APSTATS.I.C: Comparing distributions of univariate data (dotplots, back-to-back stemplots, parallel boxplots)
  1. Comparing center and spread: within group, between group variation
  2. Comparing clusters and gaps
  3. Comparing outliers and other unusual features
  4. Comparing shapes
● APSTATS.I.D: Exploring bivariate data
  1. Analyzing patterns in scatterplots
  2. Correlation and linearity
  3. Least-squares regression line
  4. Residual plots, outliers and influential points
● APSTATS.I.E: Exploring categorical data
  1. Frequency tables and bar charts
  2. Marginal and joint frequencies for two-way tables
  3. Conditional relative frequencies and association
  4. Comparing distributions using bar charts
● APSTATS.III.C: The normal distribution
1. Properties of the normal distribution
2. Using tables of the normal distribution
3. The normal distribution as a model for measurements

Supporting Standards for unit:
- ISTE-KNOWLEDGE COLLECTOR.3.C - curate information from digital resources using a variety of tools and methods to create collections of artifacts that demonstrate meaningful connections or conclusions.
- ISTE-COMPUTATIONAL THINKER.5.B - collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.
- ISTE-KNOWLEDGE COLLECTOR.3.D - build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.

Essential Questions:
1. How do you construct and interpret graphical displays of distributions of univariate data?
2. How do you summarize distributions of univariate data quantitatively?
3. How do you compare distributions of univariate data?
4. How do you explore bivariate data?
5. How do you explore categorical data?
6. How do you determine probabilities under the Normal distribution?

Enduring Understanding/Big Ideas:
1. Use appropriate display (stemplot, dotplot, histogram, etc.) to analyze distributions of univariate data.
2. Utilize appropriate measures of center and spread and concepts of outliers, shape and position to make conclusions about a data set.
3. Identify differences and similarities of distributions within a single data set and between data sets.
4. Utilize scatterplots and least-square regression lines (LSRs) to visualize the strength, direction, and form of the relationship as well as outliers within the data set.
5. Two-way tables of categorical data can be used to extract meaningful associations between two variables.
6. Utilize the standardized value equation, the z-table, and technology to determine a probability associated with a Normally distributed random variable.
### Unit Vocabulary:

<table>
<thead>
<tr>
<th>Academic Cross-Curricular Words</th>
<th>Content/Domain Specific</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qualitative</td>
<td>Nominal</td>
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<td>Quantitative</td>
<td>Ordinal</td>
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<td>Frequency</td>
<td>Discrete</td>
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<td>Symmetry</td>
<td>Continuous</td>
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<td>Outlier</td>
<td>Relative Frequency</td>
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<td>Influential</td>
<td>Distribution</td>
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<td>Percentile</td>
<td>Marginal Distribution</td>
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<td>Quartile</td>
<td>Conditional Distribution</td>
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<td>Response Variable</td>
<td>Dotplot</td>
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<td>Explanatory Variable</td>
<td>Stemplot</td>
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<td>Association</td>
<td>Histogram</td>
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<td>Extrapolation</td>
<td>Skew</td>
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<td>Range</td>
<td>Bimodal</td>
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<td>Predicted Value</td>
<td>Unimodal</td>
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<td>Mean</td>
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<td>Median</td>
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<td>Mode</td>
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<td>Interquartile Range</td>
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<td>Five-Number Summary</td>
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<td>Box Plot</td>
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<td>Standard Deviation</td>
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<td>Variance</td>
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<td>Cumulative Relative Frequency</td>
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<td>Z-score/Standardized Value</td>
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<td>Empirical Rule</td>
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<td>Density</td>
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<td></td>
<td>Scatterplot</td>
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<td></td>
<td>Correlation Coefficient</td>
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<td>Least-Squares Regression Line</td>
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<td>Residual</td>
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<td></td>
<td>Coefficient of Determination</td>
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<td></td>
<td>Standard Deviation of the Residuals</td>
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</tbody>
</table>

**Resources for Vocabulary Development:** Glossary handout with notes/flashcards
Engaging Experience 1

Title: KenKen Class Completion Times

Suggested Length of Time: 60 minutes

Standards Addressed

Priority:

- APSTATS.I.A: Constructing and interpreting graphical displays of distributions of univariate data (dotplot, stemplot, histogram, cumulative frequency plot)
  1. Center and spread
  2. Clusters and gaps
  3. Outliers and other unusual features
  4. Shape

- APSTATS.I.B: Summarizing distributions of univariate data
  1. Measuring center: median, mean
  2. Measuring spread: range, interquartile range, standard deviation
  3. Measuring position: quartiles, percentiles, standardized scores (z-scores)
  4. Using boxplots
  5. The effect of changing units on summary measures

- APSTATS.I.C: Comparing distributions of univariate data (dotplots, back-to-back stemplots, parallel boxplots)
  1. Comparing center and spread: within group, between group variation
  2. Comparing clusters and gaps
  3. Comparing outliers and other unusual features
  4. Comparing shapes

- APSTATS.III.C: The normal distribution
  1. Properties of the normal distribution
  2. Using tables of the normal distribution
  3. The normal distribution as a model for measurements

Supporting:

- ISTE-COMPUTATIONAL THINKER.5.B - collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.
**Detailed Description/Instructions:** Each student will complete an Easy 4x4 KenKen puzzle and their completion times will be make up the data set for this activity. Students will then create appropriate graphs to display the data along with appropriate numerical summaries of the data set. Using these graphs and summary statistics students will answer questions regarding the distribution of the data and their position within the data.

**Bloom’s Levels:** Evaluate

**Webb’s DOK:** 3
Topic 2: Modeling Distributions of Data

Engaging Experience 1
Title: How Likely Are You to Be Rich?
Suggested Length of Time: 10 minutes

Standards Addressed
Priority:
- APSTATS.I.E: Exploring categorical data
  1. Frequency tables and bar charts
  2. Marginal and joint frequencies for two-way tables
  3. Conditional relative frequencies and association
  4. Comparing distributions using bar charts

Supporting:
- ISTE-COMPUTATIONAL THINKER.5.B - collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.

Detailed Description/Instructions: Students will be provided with a two-way table summarizing the responses of young adults by gender and by a self-assessment of their likely financial outcomes. Students will create appropriate graphs for the data (histogram/pie chart), determine marginal and conditional frequencies relating to the data, and make comparisons across variables using the tabulated data and the graphs.

Bloom’s Levels: Apply
Webb’s DOK: 2
Engaging Experience 1

Title: Who Stole Kraviec’s Red Bull? (Using LSR to Make Predictions)

Suggested Length of Time: 60 minutes

Standards Addressed

Priority:

- APSTATS.I.D: Exploring bivariate data
  1. Analyzing patterns in scatterplots
  2. Correlation and linearity
  3. Least-squares regression line
  4. Residual plots, outliers and influential points
  5. Transformations to achieve linearity: logarithmic and power transformations

Supporting:

- ISTE-KNOWLEDGE COLLECTOR.3.D - build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.

Detailed Description/Instructions: Students will record the height and shoe size for each student in the class in order to create a LSR line which will be used to predict the height of the thief based upon a shoe print. Students will plot the data, describe the relationship, use residual plots to determine the appropriateness of using a linear model, use technology to determine the slope and intercept of the LSR line, interpret the coefficients of the LSR line in context, and make predictions using the LSR line.

Bloom’s Levels: Evaluate

Webb’s DOK: 3
Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.)

What is a better predictor of battery life in netbooks, weight or cost? What is a better predictor of the cost of a used car, age or mileage? What is a better predictor of winning percentages, points scored or points allowed?

In this project students will investigate which of two possible explanatory variables is a better predictor of a response variable by doing a thorough analysis and comparison of the relationships between each pair of variables. Students will not use examples listed - they will be required to develop their own.

The student paper should include the following components:

1. Introduction: introduce the context of the study, define the variables being investigated and discuss any preliminary hypothesis about the relationships between the variables (Explanatory Variable A, Explanatory Variable B, Response Variable, and Hypothesis need to be clearly stated).
2. Data Collection: describe how the data were obtained. Include the data in a table with at least 20 observations.
3. Graphs: display the relationships in well-labeled scatterplots, including the response variable on the same scale in each plot. Describe the relationships in each scatterplot and compare the relationships.
4. Numerical Summaries and Interpretations: calculate and interpret the correlation, equation of the least-squares regression line, and $r^2$ for each relationship. Make a residual plot for each relationship. Draw least-squares regression line on the scatterplot and clearly label on the line.
5. Conclusion and Discussion: decide which explanatory variable does a better job of predicting the response variable, citing specific evidence from the graphs and numerical summaries. Discuss when it would be appropriate to make predictions using the least-squares regression line and any potential limitations model.
## Summary of Engaging Learning Experiences for Topics

<table>
<thead>
<tr>
<th>Topic</th>
<th>Engaging Experience Title</th>
<th>Description</th>
<th>Suggested Length of Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explore Data</td>
<td>KenKen Class Completion Times</td>
<td>Each student will complete an Easy 4x4 KenKen puzzle and their completion times will be make up the data set for this activity. Students will then create appropriate graphs to display the data along with appropriate numerical summaries of the data set. Using these graphs and summary statistics students will answer questions regarding the distribution of the data and their position within the data.</td>
<td>60 minutes</td>
</tr>
<tr>
<td>Modeling Distributions of Data</td>
<td>How Likely Are You to Be Rich?</td>
<td>Students will be provided with a two-way table summarizing the responses of young adults by gender and by a self-assessment of their likely financial outcomes. Students will create appropriate graphs for the data (histogram/pie chart), determine marginal and conditional frequencies relating to the data, and make comparisons across variables using the tabulated data and the graphs.</td>
<td>10 minutes</td>
</tr>
<tr>
<td>Describing Relationships</td>
<td>Who Stole Kraviec’s Red Bull?</td>
<td>Students will record the height and shoe size for each student in the class in order to create a LSR line which will be used to predict the height of the thief based upon a shoe print. Students will plot the data, describe the relationship, use residual plots to determine the appropriateness of using a linear model, use technology to determine the slope and intercept of the LSR line, interpret the coefficients of the LSR line in context, and make predictions using the LSR line.</td>
<td>60 minutes</td>
</tr>
</tbody>
</table>
Unit 2: Sampling and Experimentation

Subject: Statistics
Grade: 11, 12
Name of Unit: Sampling and Experiment
Length of Unit: 2 weeks

Overview of Unit: Data must be collected according to a well-developed plan if valid information on a conjecture is to be obtained. This plan includes clarifying the question and deciding upon a method of data collection and analysis.

Priority Standards for unit:
- APSTATS.II.A: Overview of methods of data collection
  1. Census
  2. Sample survey
  3. Experiment
  4. Observational study
- APSTATS.II.B: Planning and conducting surveys
  1. Characteristics of a well-designed and well-conducted survey
  2. Populations, samples and random selection
  3. Sources of bias in sampling and surveys
  4. Sampling methods, including simple random sampling, stratified random sampling and cluster sampling
- APSTATS.II.C: Planning and conducting experiments
  1. Characteristics of a well-designed and well-conducted experiment
  2. Treatments, control groups, experimental units, random assignments and replication
  3. Sources of bias and confounding, including placebo effect and blinding
  4. Completely randomized design
  5. Randomized block design, including matched pairs design
- APSTATS.II.D: Generalizability of results and types of conclusions that can be drawn from observational studies, experiments and surveys

Supporting Standards for unit:
- ISTE-INNOVATIVE DESIGNER.4.A - know and use a deliberate design process for generating ideas, testing theories, creating innovative artifacts or solving authentic problems.
- TT.AB.D.7: Students will develop language and knowledge to accurately and respectfully describe how people (including themselves) are both similar to and different from each other and others in their identity groups.
- TT.AB.J.12: Students will recognize unfairness on the individual level (e.g., biased speech) and injustice at the institutional or systemic level (e.g., discrimination).
• TT.AB.J.13: Students will analyze the harmful impact of bias and injustice on the world, historically and today.

**Essential Questions:**

1. How do we collect data in order to draw meaningful conclusions about a population or the effect of a treatment?
2. How can we plan and conduct surveys that will give data representative of the population?
3. How can we plan and conduct experiments that will allow us to make a fair comparison of different treatments?
4. How do we select the appropriate method of data collection that will allow us to generalize our results to the greater population or to determine causality?

**Enduring Understanding/Big Ideas:**

1. Utilize different survey methods (census/sampling) and experimental designs (assigned treatments/observational) in order to generate samples that are representative of the population.
2. Utilize different methods of random selection to generate samples that are representative of different populations and minimize sources of potential bias.
3. Utilize randomization, blocking, and control in experimental design to allow us to make fair comparisons and minimize bias.
4. Randomization in the selection of the sample and in the assignment of treatments allows us to generalize results to the greater population and make conclusions about cause and effect.
## Unit Vocabulary:

<table>
<thead>
<tr>
<th>Academic Cross-Curricular Words</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>Selection Bias</td>
</tr>
<tr>
<td>Sample</td>
<td>Convenience Sample</td>
</tr>
<tr>
<td>Bias</td>
<td>Voluntary Response Sample</td>
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<tr>
<td>Inference</td>
<td>Undercoverage</td>
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<tr>
<td>Observational Study</td>
<td>Response Bias</td>
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<tr>
<td>Experiment</td>
<td>Survey Bias</td>
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<tr>
<td>Treatment</td>
<td>Random Sampling</td>
</tr>
<tr>
<td>Factor</td>
<td>Simple Random Sample</td>
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<td>Experimental Units</td>
<td>Stratified Random Sample</td>
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<tr>
<td>Subjects</td>
<td>Cluster Sample</td>
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<tr>
<td>Random Assignment</td>
<td>Confounding</td>
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<td>Control Group</td>
<td>Lurking</td>
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<td>Replication</td>
<td>Statistically Significant</td>
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<tr>
<td>Placebo</td>
<td>Blocking</td>
</tr>
<tr>
<td>Double-Blind</td>
<td>Matched Pairs</td>
</tr>
<tr>
<td>Causation</td>
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</tr>
</tbody>
</table>

**Resources for Vocabulary Development:** Glossary handout with notes/flashcards
Topic 1: Sampling

**Engaging Experience 1**

**Title:** Sampling Methods at a Large University (AP FRQ #2, 2013)

**Suggested Length of Time:** 15 minutes

**Standards Addressed**

*Priority:*

- APSTATS.II.A: Overview of methods of data collection
  1. Census
  2. Sample survey
- APSTATS.II.B: Planning and conducting surveys
  1. Characteristics of a well-designed and well-conducted survey
  2. Populations, samples and random selection
  3. Sources of bias in sampling and surveys
  4. Sampling methods, including simple random sampling, stratified random sampling and cluster sampling

*Supporting:*

- ISTE-INNOVATIVE DESIGNER.4.A - know and use a deliberate design process for generating ideas, testing theories, creating innovative artifacts or solving authentic problems.
- TT.AB.D.7: Students will develop language and knowledge to accurately and respectfully describe how people (including themselves) are both similar to and different from each other and others in their identity groups.
- TT.AB.J.12: Students will recognize unfairness on the individual level (e.g., biased speech) and injustice at the institutional or systemic level (e.g., discrimination).
- TT.AB.J.13: Students will analyze the harmful impact of bias and injustice on the world, historically and today.

**Detailed Description/Instruction:** Students will consider the situation of a school administrator attempting to obtain a representative sample of 500 students from a very large university. Students will consider potential sources of bias, random sampling methods, stratified sampling methods, and how different sampling methods can provide more precise point estimates.

**Bloom’s Levels:** Evaluate

**Webb’s DOK:** 3
Engaging Experience 1
Title: Shampoo Formula Comparison (AP FRQ #2, 2004)

Suggested Length of Time: 15 minutes

Standards Addressed

Priority:

- APSTATS.II.A: Overview of methods of data collection
  3. Experiment
  4. Observational study
- APSTATS.II.C: Planning and conducting experiments
  1. Characteristics of a well-designed and well-conducted experiment
  2. Treatments, control groups, experimental units, random assignments and replication
  3. Sources of bias and confounding, including placebo effect and blinding
  4. Completely randomized design
  5. Randomized block design, including matched pairs design
- APSTATS.II.D: Generalizability of results and types of conclusions that can be drawn from observational studies, experiments and surveys

Supporting:

- ISTE-INNOVATIVE DESIGNER.4.A - know and use a deliberate design process for generating ideas, testing theories, creating innovative artifacts or solving authentic problems.
- TT.AB.D.7: Students will develop language and knowledge to accurately and respectfully describe how people (including themselves) are both similar to and different from each other and others in their identity groups.
- TT.AB.J.12: Students will recognize unfairness on the individual level (e.g., biased speech) and injustice at the institutional or systemic level (e.g., discrimination).
- TT.AB.J.13: Students will analyze the harmful impact of bias and injustice on the world, historically and today.

Detailed Description/Instructions: Students will consider the case of a group of researchers who are looking to compare the efficacy of a new and old shampoo formula. Students will determine the relative merits of a fully randomized experiment versus a blocking experiment depending on which variables the experimenters believe to be explanatory. Students will draw conclusions as to the generalizability of the experiment based upon how the participants were selected and how treatments were assigned.

Bloom’s Levels: Evaluate

Webb’s DOK: 3

Rubric: to be created
Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will complete a unit test composed of Secured AP Multiple Choice (20) and Free Response (3) questions with time constraints and grading rubric similar to that of the official AP Exam. As the culminating experience in AP Statistics is the AP Statistics Exam administered in May, it follows that the best preparation would be questions of the same form and style as what they will see on the official exam.

Rubric for Engaging Scenario: refer to the Secured AP Scoring Guide
### Summary of Engaging Learning Experiences for Topics

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<tbody>
<tr>
<td>Sampling</td>
<td>Sampling Methods at a Large University</td>
<td>Students will consider the situation of a school administrator attempting to obtain a representative sample of 500 students from a very large university. Students will consider potential sources of bias, random sampling methods, stratified sampling methods, and how different sampling methods can provide more precise point estimates.</td>
<td>15 minutes</td>
</tr>
<tr>
<td>Experimenting</td>
<td>Shampoo Formula Comparison</td>
<td>Students will consider the case of a group of researchers who are looking to compare the efficacy of a new and old shampoo formula. Students will determine the relative merits of a fully randomized experiment versus a blocking experiment depending on which variables the experimenters believe to be explanatory. Students will draw conclusions as to the generalizability of the experiment based upon how the participants were selected and how treatments were assigned.</td>
<td>15 minutes</td>
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Unit 3: Anticipating Patterns

**Subject:** Statistics

**Grade:** 11, 12

**Name of Unit:** Anticipating Patterns

**Length of Unit:** 5 weeks

**Overview of Unit:** Probability is the tool used for anticipating what the distribution of data should look like under a given model. Random phenomena are not haphazard: they display an order that emerges only in the long run and is described by a distribution. The mathematical description of variation is central to statistics. The probability required for statistical inference is not primarily axiomatic or combinatorial but is oriented toward using probability distributions to describe data.

**Priority Standards for unit:**

- **APSTATS.III.A: Probability**
  1. Interpreting probability, including long-run relative frequency interpretation
  2. “Law of Large Numbers” concept
  3. Addition rule, multiplication rule, conditional probability and independence
  4. Discrete random variables and their probability distributions, including binomial and geometric
  5. Simulation of random behavior and probability distributions
  6. Mean (expected value) and standard deviation of a random variable, and linear transformation of random variable

- **APSTATS.III.B: Combining independent random variables**
  1. Notion of independence versus dependence
  2. Mean and standard deviation for sums and differences of independent random variables

- **APSTATS.III.C: The normal distribution**
  1. Properties of the normal distribution
  2. Using tables of the normal distribution
  3. The normal distribution as a model for measurements

- **APSTATS.III.D: Sampling distributions**
  1. Sampling distribution of a sample proportion
  2. Sampling distribution of a sample mean
  3. Central Limit Theorem
  4. Simulation of sampling distributions
  5. T-distribution
Supporting Standards for unit:
- ISTE-KNOWLEDGE COLLECTOR.3.D - build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.
- ISTE-COMPUTATIONAL THINKER.5.B - collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.

Essential Questions:
1. How do you calculate and interpret probability?
2. How do you combine independent random variables?
3. How is the Normal distribution used to model data sets?
4. How are sampling distributions interpreted and applied to a population?

Enduring Understanding/Big Ideas:
1. Probability models assign probabilities to events through rules and formulas that must be appropriately applied.
2. A random variable summarizes the outcomes of a chance process and provides a probability distribution that can be transformed and combined with other variables.
3. For certain sets of data, a Normal distribution provides a density curve to interpret positions of data points and probabilities of events.
4. Using the Central Limit Theorem, the mean and proportion sampling distributions provide unbiased estimators of the corresponding population parameters.

Unit Vocabulary:

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<thead>
<tr>
<th>Academic Cross-Curricular Words</th>
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<tbody>
<tr>
<td>Venn-diagram</td>
<td>Law of large numbers</td>
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<td>Simulation</td>
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<td>Sample space</td>
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<td>Event</td>
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<td>Complement</td>
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<td>Mutually exclusive (disjoint)</td>
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<td>Two-way table</td>
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<td>Union</td>
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<td>Intersection</td>
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<td>Conditional probability</td>
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<td>Independent</td>
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<td>Tree diagram</td>
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<td>Conditional probability</td>
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<td>Random variable</td>
<td>Expected value</td>
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<td>Linear transformation</td>
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<td>Independent random variable</td>
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<td>Binomial random variable</td>
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<td>Geometric random variable</td>
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<td>Normal distribution</td>
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<td>68-95-99.7 rule</td>
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<td>Parameter</td>
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<td>Statistic</td>
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<td>Population distribution</td>
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<td>Sampling distribution</td>
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<td>Unbiased estimator</td>
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<td>Biased estimator</td>
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<tr>
<td>Variability</td>
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<tr>
<td>Central Limit Theorem</td>
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</tbody>
</table>

**Resources for Vocabulary Development:** Glossary handout with notes/flashcards
**Engaging Experience 1**

**Title:** Investigating Randomness

**Suggested Length of Time:** 30 minutes

**Standards Addressed**

*Priority:*

- APSTATS.III.A: Probability
  1. Interpreting probability, including long-run relative frequency interpretation
  2. “Law of Large Numbers” concept
  5. Simulation of random behavior and probability distributions

*Supporting:*

- ISTE-COMPUTATIONAL THINKER.5.B - collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.

**Detailed Description/Instructions:** Students will simulate tossing a coin with an applet. They will toss a coin 10 times, 50 times, then 100 times. Students construct a frequency graph that illustrates the larger number of trials produce a proportion closer to 0.50.

**Bloom’s Levels:** Understand

**Webb’s DOK:** 2

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**Engaging Experience 2**

**Title:** Venn diagrams, Two-way tables, and Probability

**Suggested Length of Time:** 20 minutes

**Standards Addressed**

*Priority:*

- APSTATS.III.A: Probability
  3. Addition rule, multiplication rule, conditional probability and independence

**Detailed Description/Instructions:** Students are provided with a two-way table. They will construct a Venn-diagram that represents the outcomes and compute the probabilities associated with these outcomes.

**Bloom’s Levels:** Understand

**Webb’s DOK:** 2
**Engaging Experience 1**

**Title:** Auto Dealership

**Suggested Length of Time:** Two-45 minute sessions

**Standards Addressed**

**Priority:**

- APSTATS.III.A: Probability
  - 4. Discrete random variables and their probability distributions, including binomial and geometric
  - 6. Mean (expected value) and standard deviation of a random variable, and linear transformation of random variable
- APSTATS.III.B: Combining independent random variables
  - 7. Notion of independence

**Supporting:**

- ISTE-KNOWLEDGE COLLECTOR.3.D - build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.

**Detailed Description/Instructions:** Students are given a probability distribution of the number of cars sold at a dealership and calculate the mean and standard deviation of the random variable. Students produce another random variable that models a $500 bonus added to each car sold. Students compute the mean and standard deviation of this new random variable. Students are then provided a new random variable that gives the number of cars leased and find and interpret the average and standard deviation of the sum of these random variables.

**Bloom’s Levels:** Apply

**Webb’s DOK:** 2
Engaging Experience 1

Title: The Candy Machine

Suggested Length of Time: 45 minutes

Standards Addressed

Priority:

- APSTATS.III.D: Sampling distributions
  1. Sampling distribution of a sample proportion
  6. Simulation of sampling distributions

Supporting:

- ISTE-COMPUTATIONAL THINKER.5.B - collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.

Detailed Description/Instructions: Using an applet, students simulate a Reese’s Pieces candy machine to investigate the sample-to-sample variability in the proportion of orange candies dispensed by the machine. Student take sample of varying sizes. For each sample size, they produce a dot plot of the proportion of orange candies. Students describe the changes in mean and standard deviation of the distributions for each sample size.

Bloom’s Levels: Apply

Webb’s DOK: 3
Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will complete a unit test composed of Secured AP Multiple Choice (20) and Free Response (3) questions with time constraints and grading rubric similar to that of the official AP Exam. As the culminating experience in AP Statistics is the AP Statistics Exam administered in May, it follows that the best preparation would be questions of the same form and style as what they will see on the official exam.

**Rubric for Engaging Scenario:** refer to the Secured AP Scoring Guide
### Summary of Engaging Learning Experiences for Topics

<table>
<thead>
<tr>
<th>Topic</th>
<th>Engaging Experience Title</th>
<th>Description</th>
<th>Suggested Length of Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>Investigating Randomness</td>
<td>Students will simulate tossing a coin with an applet. They will toss a coin 10 times, 50 times, then 100 times. Students construct a frequency graph that illustrates the larger number of trials produce a proportion closer to 0.50.</td>
<td>30 minutes</td>
</tr>
<tr>
<td>Probability</td>
<td>Venn Diagrams, Two-way Tables, and Probability</td>
<td>Students are provided with a two-way table. They will construct a Venn-diagram that represents the outcomes and compute the probabilities associated with these outcomes.</td>
<td>20 minutes</td>
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<tr>
<td>Random Variables</td>
<td>Auto Dealership</td>
<td>Students are given a probability distribution of the number of cars sold at a dealership and calculate the mean and standard deviation of the random variable. Students produce another models a $500 bonus added to each car sold. Students compute the mean and standard deviation of this new random variable. Students are then provided a new random variable that gives the number of cars leased and find and interpret the average and standard deviation of the sum of these random variables.</td>
<td>20 - 45 minute sessions</td>
</tr>
<tr>
<td>Sampling Distributions</td>
<td>The Candy Machine</td>
<td>Using an applet, students simulate a Reese’s Pieces candy machine to investigate the sample-to-sample variability. Student take sample of varying sizes and they produce a dot plot of the proportion of orange candies. Students describe the changes in mean and standard deviation of the distributions for each sample size.</td>
<td>45 minutes</td>
</tr>
</tbody>
</table>
Unit 4: Statistical Inference

Subject: Statistics
Grade: 11, 12
Name of Unit: Statistical Inference
Length of Unit: 7 weeks

Overview of Unit: Statistical inference guides the selection of appropriate models. Models and data interact in statistical work: models are used to draw conclusions from data, while the data are allowed to criticize and even falsify the model through inferential and diagnostic methods. Inference from data can be thought of as the process of selecting a reasonable model, including a statement in probability language, of how confident one can be about the selection. The optional activities included in this unit may be used when time allows and are designed to fill a portion of the gap between Statistics and AP Statistics.

Priority Standards for unit:
- APSTATS.IV.A: Estimation (point estimators and confidence intervals)
  1. Estimating population parameters and margins of error
  2. Properties of point estimators, including unbiasedness and variability
  3. Logic of confidence intervals, meaning of confidence level and confidence intervals, and properties of confidence intervals
  4. Large sample confidence interval for a proportion
  5. Large sample confidence interval for a difference between two proportions
  6. Confidence interval for a mean
  7. Confidence interval for a difference between two means (unpaired and paired)
  8. Confidence interval for the slope of a least-squares regression line
- APSTATS.IV.B: Tests of significance
  1. Logic of significance testing, null and alternative hypotheses; p-values; one- and two-sided tests; concepts of Type I and Type II errors; concept of power
  2. Large sample test for a proportion
  3. Large sample test for a difference between two proportions
  4. Test for a mean
  5. Test for a difference between two means (unpaired and paired)

Supporting Standards for unit:
- ISTE-KNOWLEDGE COLLECTOR.3.D - build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.
- ISTE-COMPUTATIONAL THINKER.5.B - collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.
Essential Questions:
1. How do we estimate population proportions, population means, and differences population parameters?
2. How are tests of significance used as evidence for some claim about a parameter?

Enduring Understanding/Big Ideas:
1. The statistics from the sample data gives us a point estimate around which we can generate a confidence interval using the properties of the sampling distribution of that sample statistic.
2. After certain criteria are met, a significance test can be performed to either reject or fail to reject a null hypothesis.

<table>
<thead>
<tr>
<th>Academic Cross-Curricular Words</th>
<th>Content/Domain Specific</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape</td>
<td>Point estimator</td>
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<td>Center</td>
<td>Confidence interval</td>
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<td>Spread</td>
<td>Margin of error</td>
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<td>Standard error</td>
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<td>Degrees of freedom</td>
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<td>One-sample t interval</td>
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<td>Robust</td>
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<td>Significance test</td>
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<td>Alternative hypothesis</td>
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<td>P-value</td>
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<td>Significance level</td>
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<td>Statistically significant</td>
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<td>Type I error</td>
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<td>Type II error</td>
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<td>Power</td>
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<td>Test statistic</td>
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<td>One-sample t-statistic</td>
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<td>Paired data</td>
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<td>Two-sample z interval</td>
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<td>Significance tests</td>
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<td>Pooled sample proportion</td>
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<td>Two-sample t statistic</td>
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</tbody>
</table>

Resources for Vocabulary Development: Glossary handout with notes/ Flashcards
Engaging Experience 1
Title: Social Security Satisfaction (AP FRQ #4, 2002)
Suggested Length of Time: 15 minutes
Standards Addressed
Priority:
- APSTATS.IV.A: Estimation (point estimators and confidence intervals)
  1. Estimating population parameters and margins of error
  2. Properties of point estimators, including unbiasedness and variability
  3. Logic of confidence intervals, meaning of confidence level and confidence intervals, and properties of confidence intervals
  4. Large sample confidence interval for a proportion

Supporting:
- ISTE-KNOWLEDGE COLLECTOR.3.D - build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.

Detailed Description/Instructions: Given a set of survey results regarding public attitudes of the Social Security system, students will check that the conditions for generating a confidence interval have been satisfied, generate a confidence interval for the proportion of adults who responded “Make some major changes”, and interpret the confidence interval and confidence level in the context of the survey.

Bloom’s Levels: Evaluate
Webb’s DOK: 3

Engaging Experience 2
Title: Bird Watching Scores
Suggested Length of Time: 10 minutes
Standards Addressed
Priority:
- APSTATS.IV.A: Estimation (point estimators and confidence intervals)
  1. Estimating population parameters and margins of error
  2. Properties of point estimators, including unbiasedness and variability
  3. Logic of confidence intervals, meaning of confidence level and confidence intervals, and properties of confidence intervals
  4. Confidence interval for a mean

Supporting:
- ISTE-COMPUTATIONAL THINKER.5.B - collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.
**Detailed Description/Instructions:** Students will be given a data set containing the bird watching scores for all of the bird watchers involved in a university study. Students will check that the data set meets the conditions necessary for creating a one-sample confidence interval for the mean bird watching scores. Difficulties will arise around how to check the assumption of approximate Normality of the data. Once all conditions have been checked, students will generate a confidence interval for the mean bird watching score and interpret that score in the context of the study.

**Bloom’s Levels:** Analyze

**Webb’s DOK:** 2
Engaging Experience 1
Title: Better Batteries
Suggested Length of Time: 45 minutes

Standards Addressed

Priority:
- APSTATS.IV.B: Tests of significance
  1. Logic of significance testing, null and alternative hypotheses; p-values; one- and two-sided tests; concepts of Type I and Type II errors; concept of power
  4. Test for a mean

Detailed Description/Instructions: Students are provided with the simple random sample of the lifetime of 15 batteries. Students check the random, Normal and independence conditions to perform a significance test. They write a null and alternate hypothesis and calculate the test statistic.

Bloom’s Levels: Apply
Webb’s DOK: 3
Topic 3: Comparing Two Populations

Engaging Experience 1 - OPTIONAL: If time allows
Title: CPR Effectiveness (AP FRQ #3, 2009)
Suggested Length of Time: 15 minutes

Standards Addressed
Priority:

- APSTATS.IV.A: Estimation (point estimators and confidence intervals)
  1. Estimating population parameters and margins of error
  2. Properties of point estimators, including unbiasedness and variability
  3. Logic of confidence intervals, meaning of confidence level and confidence intervals, and properties of confidence intervals
  5. Large sample confidence interval for a difference between two proportions

Supporting:
- ISTE-KNOWLEDGE COLLECTOR.3.D - build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.

Detailed Description/Instructions: Students will be given a medical study considering the effectiveness of two methods for saving lives of heart attack victims. Students will generate point estimates for the survival rates of the two methods, check that the conditions for creating a confidence interval have been satisfied, form a confidence interval, and interpret that confidence interval (and confidence level) in the context of the study. Students will need to understand the logic of confidence intervals to determine whether there is a difference in the efficacies of these two methods (i.e. is 0 contained in the confidence interval?).

Bloom’s Levels: Evaluate
Webb’s DOK: 3

Engaging Experience 2 - OPTIONAL: If time allows
Title: Cholesterol Drug Efficacy (AP FRQ #5, 2007)
Suggested Length of Time: 15 minutes

Standards Addressed
Priority:

- APSTATS.IV.A: Estimation (point estimators and confidence intervals)
  1. Estimating population parameters and margins of error
  2. Properties of point estimators, including unbiasedness and variability
  3. Logic of confidence intervals, meaning of confidence level and confidence intervals, and properties of confidence intervals
  7. Confidence interval for a difference between two means (unpaired and paired)
Supporting:
- ISTE-KNOWLEDGE COLLECTOR.3.D - build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.

**Detailed Description/Instructions:** Students will consider a medical study comparing the mean cholesterol drop for participants taking one of two cholesterol-lowering medications. Students will generate point estimates for the mean cholesterol drop of each of the drugs, check the conditions for generating a confidence interval for the difference of the mean cholesterol drops, form the confidence interval, and interpret the confidence interval in the context of the study. Again, students will use the logic of confidence intervals to determine whether there is significant statistical evidence to suggest that there is a difference in the mean cholesterol drops of the two drugs.

**Bloom’s Levels:** Evaluate

**Webb’s DOK:** 3
Engaging Scenario

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will complete a unit test composed of Secured AP Multiple Choice (20) and Free Response (3) questions with time constraints and grading rubric similar to that of the official AP Exam. As the culminating experience in AP Statistics is the AP Statistics Exam administered in May, it follows that the best preparation would be questions of the same form and style as what they will see on the official exam.

**Rubric for Engaging Scenario:** refer to the Secured AP Scoring Guide
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<tr>
<td>Confidence Interval</td>
<td>Social Security Satisfaction</td>
<td>Given a set of survey results regarding public attitudes of the Social Security system, students will check that the conditions for generating a confidence interval have been satisfied, generate a confidence interval for the proportion of adults who responded “Make some major changes”, and interpret the confidence interval and confidence level in the context of the survey.</td>
<td>15 minutes</td>
</tr>
<tr>
<td>Confidence Interval</td>
<td>Bird Watching Scores</td>
<td>Students will be given a data set containing the bird watching scores for all of the bird watchers involved in a university study. Students will check that the data set meets the conditions necessary for creating a one-sample confidence interval for the mean bird watching scores. Difficulties will arise around how to check the assumption of approximate Normality of the data. Once all conditions have been checked, students will generate a confidence interval for the mean bird watching score and interpret that score in the context of the study.</td>
<td>10 minutes</td>
</tr>
<tr>
<td>Significance Tests</td>
<td>Better Batteries</td>
<td>Students are provided with the simple random sample of the lifetime of 15 batteries. Students check the random, Normal and independence conditions to perform a significance test. They write a null and alternate hypothesis and calculate the test statistic.</td>
<td>45 minutes</td>
</tr>
<tr>
<td>Comparing Two Populations</td>
<td>CPR Effectiveness</td>
<td>Students will be given a medical study considering the effectiveness of two methods for saving lives of heart attack victims. Students will generate point estimates for the survival rates of the two methods, check that the conditions for</td>
<td>15 minutes</td>
</tr>
<tr>
<td><strong>OPTIONAL</strong></td>
<td><strong>Comparing Two Populations</strong></td>
<td><strong>Cholesterol Drug Efficacy</strong></td>
<td>Students will consider a medical study comparing the mean cholesterol drop for participants taking one of two cholesterol-lowering medications. Students will generate point estimates for the mean cholesterol drop of each of the drugs, check the conditions for generating a confidence interval for the difference of the mean cholesterol drops, form the confidence interval, and interpret the confidence interval in the context of the study. Again, students will use the logic of confidence intervals to determine whether there is significant statistical evidence to suggest that there is a difference in the mean cholesterol drops of the two drugs.</td>
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Creating a confidence interval have been satisfied, form a confidence interval, and interpret that confidence interval (and confidence level) in the context of the study. Students will need to understand the logic of confidence intervals to determine whether there is a difference in the efficacies of these two methods (i.e. is 0 contained in the confidence interval?).
Unit of Study Terminology

**Appendices:** All Appendices and supporting material can be found in this course’s shell course in the District’s Learning Management System.

**Assessment Leveling Guide:** A tool to use when writing assessments in order to maintain the appropriate level of rigor that matches the standard.

**Big Ideas/Enduring Understandings:** Foundational understandings teachers want students to be able to discover and state in their own words by the end of the unit of study. These are answers to the essential questions.

**Engaging Experience:** Each topic is broken into a list of engaging experiences for students. These experiences are aligned to priority and supporting standards, thus stating what students should be able to do. An example of an engaging experience is provided in the description, but a teacher has the autonomy to substitute one of their own that aligns to the level of rigor stated in the standards.

**Engaging Scenario:** This is a culminating activity in which students are given a role, situation, challenge, audience, and a product or performance is specified. Each unit contains an example of an engaging scenario, but a teacher has the ability to substitute with the same intent in mind.

**Essential Questions:** Engaging, open-ended questions that teachers can use to engage students in the learning.

**Priority Standards:** What every student should know and be able to do. These were chosen because of their necessity for success in the next course, the state assessment, and life.

**Supporting Standards:** Additional standards that support the learning within the unit.

**Topic:** These are the main teaching points for the unit. Units can have anywhere from one topic to many, depending on the depth of the unit.

**Unit of Study:** Series of learning experiences/related assessments based on designated priority standards and related supporting standards.

**Unit Vocabulary:** Words students will encounter within the unit that are essential to understanding. Academic Cross-Curricular words (also called Tier 2 words) are those that can be found in multiple content areas, not just this one. Content/Domain Specific vocabulary words are those found specifically within the content.

**Symbols:**
- This symbol depicts an experience that can be used to assess a student’s 21st Century Skills using the rubric provided by the district.
- This symbol depicts an experience that integrates professional skills, the development of professional communication, and/or the use of professional mentorships in authentic classroom learning activities.