



# Park Hill School District

Building Successful Futures • Each Student • Every Day

## High School Math Analysis Curriculum

**Course Description:** A Pre-Calculus course for the serious and motivated college-bound student. Concentration is on analyzing problems and applying mathematical concepts introduced in Algebra II. This course is primarily taught through lecture, small group activities and projects dealing with real-life situations. The content includes a strong emphasis on logarithmic, exponential, and polynomial functions, conics, sequences and series. \*Graphing calculators are required. See instructor for recommendations.

### Scope and Sequence:

Timeframe	Unit	Instructional Topics
10-12 class periods	Fundamentals and Functions	Topic 1: Exponents and Radicals Topic 2: Variation Topic 3: Functions and Their Graphs Topic 4: Operations on Functions
10 class periods	Polynomial and Rational Functions	Topic 1: Polynomial Functions Topic 2: Rational Functions
12 class periods	Exponential and Logarithmic Functions	Topic 1: Graphing Topic 2: Logs Topic 3: Solving Equations Topic 4: Applications
5-6 class periods	Sequences and Series	Topic 1: General Sequences and Series Topic 2: Arithmetic Sequences and Series Topic 3: Geometric Sequences and Series
7-8 class periods	Conics	Topic 1: Conics

10-12 class periods	Limits	Topic 1: Limits Topic 2: Applications
7-8 class periods	Systems	Topic 1: Systems Topic 2: Partial Fractions
7-10 class periods	Trigonometry	Topic 1: Solving Trigonometric Equations Topic 2: Polar Coordinates and Graphing Topic 3: Parametric Equations

## Unit 1: Fundamentals and Functions

**Subject:** Math Analysis

**Grade:** 10, 11, 12

**Name of Unit:** Fundamentals and Functions

**Length of Unit:** 10-12 class periods

**Overview of Unit:** In this unit students will spend time reviewing concepts that will be critical to their success in this course. Those concepts include radicals, exponents, variation, parent functions, transformations, and operations on functions.

### Priority Standards for unit:

- Alg2.NQ.A.2: Create and recognize equivalent expressions involving radical and exponential forms of expressions.
- Alg2.BF.A.1: Create new functions by applying the four arithmetic operations and composition of functions (modifying the domain and range as necessary).
- Alg2.BF.A.2: Derive inverses of functions, and compose the inverse with the original function to show that the functions are inverses.
- Alg2.BF.A.3: Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for linear, quadratic, cubic, square and cube root, absolute value, exponential and logarithmic functions.
- MW.MA.120.1: Analyze problems and identify which algebraic methods and formulas are needed and solve the problem.

### Supporting Standards for unit:

- MW.MA.110.1: Identify and use algebraic terminology and function notation.
- MW.MA.120.4: Solve algebraic equations and inequalities.
- MW.MA.120.9: Interpret and analyze graphical representations to solve problems.
- MW.MA.120.10: Interpret and recognize reasonable answers.
- ISTE-INNOVATIVE DESIGNER.4.D - exhibit a tolerance for ambiguity, perseverance and the capacity to work with open-ended problems.
- ISTE-COMPUTATIONAL THINKER.5.C - break problems into component parts, extract key information, and develop descriptive models to understand complex systems or facilitate problem-solving.

Unwrapped Concepts (Students need to know)	Unwrapped Skills (Students need to be able to do)	Bloom's Taxonomy Levels	Webb's DOK
and recognize equivalent expressions involving radical and exponential forms of expressions.	Create	Understand	2

Create and equivalent expressions involving radical and exponential forms of expressions.	Recognize	Understand	2
Create new functions by the four arithmetic operations and composition of functions (modifying the domain and range as necessary).	Applying	Apply	2
inverses of functions, and compose the inverse with the original function to show that the functions are inverses.	Derive	Apply	2
Derive inverses of functions, and the inverse with the original function to show that the functions are inverses.	Compose	Apply	2
the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for linear, quadratic, cubic, square and cube root, absolute value, exponential and logarithmic functions.	Describe	Analyze	3
problems and identify which algebraic methods and formulas are needed and solve the problem.	Analyze	Analyze	3
Analyze problems and which algebraic methods and formulas are needed and solve the problem.	Identify	Analyze	2

### **Essential Questions:**

1. How are exponents and radicals related?
2. How are different forms of variation modeled?
3. Why is knowledge of parent functions useful in graphing?
4. How are operations of parent functions performed?

### **Enduring Understanding/Big Ideas:**

1. Radicals can be rewritten in exponential form and simplified.
2. Variation models include direct, inverse, and joint.
3. Parent functions can be quickly graphed using transformations.
4. Operations on parent functions are performed using addition, subtraction, multiplication, division, composition, and finding inverses.

**Unit Vocabulary:**

Academic Cross-Curricular Words	Content/Domain Specific
	Exponents Radicals Variation Function Rate of Change Composition Transformations Inverse One-to-one

**Resources for Vocabulary Development:** textbook

## Topic 1: Exponents and Radicals

### **Engaging Experience 1**

**Title:** Radicals and Exponents with White Boards

**Suggested Length of Time:** 15 minutes

#### **Standards Addressed**

*Priority:*

- Alg2.NQ.A.2: Create and recognize equivalent expressions involving radical and exponential forms of expressions.

*Supporting:*

- MW.MA.120.10: Interpret and recognize reasonable answers.

**Detailed Description/Instructions:** Students will use whiteboards to work teacher presented problems. Student will hold up their boards to show their answers so the teacher can give feedback.

**Bloom's Levels:** Understand

**Webb's DOK:** 2

## Topic 2: Variation

### **Engaging Experience 1**

**Title:** Data Extrapolation

**Suggested Length of Time:** 15 minutes

#### **Standards Addressed**

*Priority:*

- MW.MA.120.1: Analyze problems and identify which algebraic methods and formulas are needed and solve the problem.

*Supporting:*

- MW.MA.120.4: Solve algebraic equations and inequalities.

**Detailed Description/Instructions:** Students will work on getting a better understanding of the relationship between variables. Students will be given a scenario and will find an equation and determine future data points using that equation. Potential scenarios could include applying gas laws, or the gravitational force between two masses, or scenarios involving movement and examining acceleration or deceleration.

**Bloom's Levels:** Analyze

**Webb's DOK:** 3

## Topic 3: Functions and Their Graphs

### **Engaging Experience 1**

**Title:** Graphing Parent Function and Transforming Them

**Suggested Length of Time:** ½ of a class period

#### **Standards Addressed**

*Priority:*

- Alg2.BF.A.3: Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for linear, quadratic, cubic, square and cube root, absolute value, exponential and logarithmic functions.

*Supporting:*

- MW.MA.120.9: Interpret and analyze graphical representations to solve problems
- ISTE-COMPUTATIONAL THINKER.5.C - break problems into component parts, extract key information, and develop descriptive models to understand complex systems or facilitate problem-solving.

**Detailed Description/Instructions:** Students will be given the set of parent functions (linear, quadratic, cubic, radical, absolute value, piecewise, and greatest integer) and asked to create a graph for each one. They will then be asked to use their knowledge of transformations (shifts, stretches/compressions, and reflections) to create new graphs with those functions.

**Bloom's Levels:** Analyze

**Webb's DOK:** 3



## Topic 4: Operations on Functions

### **Engaging Experience 1**

**Title:** Composing Functions and Finding Inverses

**Suggested Length of Time:**  $\frac{1}{2}$  of a class period

**Standards Addressed**

*Priority:*

- Alg2.BF.A.1: Create new functions by applying the four arithmetic operations and composition of functions (modifying the domain and range as necessary).
- Alg2.BF.A.2: Derive inverses of functions, and compose the inverse with the original function to show that the functions are inverses.

*Supporting:*

- MW.MA.110.1: Identify and use algebraic terminology and function notation.

**Detailed Description/Instructions:** Students will participate in guided practice with problems involving finding inverses and composing functions.

**Bloom's Levels:** Apply

**Webb's DOK:** 2

## Engaging Scenario

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will be broken into groups and will use the Function Carnival application on Desmos to model various piecewise functions. Students will produce a graph and set of equations that model the scenario set before them. Their findings will be shared with the class via a brief presentation.

## Summary of Engaging Learning Experiences for Topics

Topic	Engaging Experience Title	Description	Suggested Length of Time
Exponents and Radicals	Radicals and Exponents with Whiteboards	Students will use whiteboards to work teacher presented problems. Student will hold up their boards to show their answers so the teacher can give feedback.	15 minutes
Variation	Data Extrapolation	Students will work on getting a better understanding of the relationship between variables. Students will be given a scenario and will find an equation and determine future data points using that equation. Potential scenarios could include applying gas laws, or the gravitational force between two masses, or scenarios involving movement and examining acceleration or deceleration.	15 minutes
Functions and Their Graphs	Graphing Parent Function and Transforming Them	Students will be given the set of parent functions (linear, quadratic, cubic, radical, absolute value, piecewise, and greatest integer) and asked to create a graph for each one. They will then be asked to use their knowledge of transformations (shifts, stretches/compressions, and reflections) to create new graphs with those functions.	½ of a class period
Operations on Functions	Composing Functions and Finding Inverses	Students will participate in guided practice with problems involving finding inverses and composing functions.	½ of a class period

## Unit 2: Polynomial and Rational Functions

**Subject:** Math Analysis

**Grade:** 10, 11, 12

**Name of Unit:** Polynomial and Rational Functions

**Length of Unit:** 10 class periods

**Overview of Unit:** In this unit students will graph polynomial functions and find the zeros. Students will graph rational functions using asymptotes and intercepts. Inequalities of both functions will be explored.

### Priority Standards for unit:

- Alg2.NQ.B.3: Know and apply the Fundamental Theorem of Algebra.
- Alg2.APR.A.2: Understand the Remainder Theorem and use it to solve problems.
- Alg2.APR.A.5: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to sketch the function defined by the polynomial.
- Alg2.REI.A.2: Solve rational equations where numerators and denominators are polynomials and where extraneous solutions may result.
- MW.MA.120.5: Solve for real and complex roots of polynomials.
- GA.MM4A1. Students will explore rational functions.
  - a. Investigate and explain characteristics of rational functions, including domain, range, zeros, points of discontinuity, intervals of increase and decrease, rates of change, local and absolute extrema, symmetry, asymptotes, and end behavior.
  - b. Find inverses of rational functions, discussing domain and range, symmetry, and function composition.
  - c. Solve rational equations and inequalities analytically, graphically, and by using appropriate technology.

### Supporting Standards for unit:

- MW.MA.120.3: Perform mathematical operations on complex numbers.
- ISTE-EMPOWERED LEARNER1.C - use technology to seek feedback that informs and improves their practice and to demonstrate their learning in a variety of ways.
- ISTE-COMPUTATIONAL THINKER.5.C - break problems into component parts, extract key information, and develop descriptive models to understand complex systems or facilitate problem-solving.

Unwrapped Concepts (Students need to know)	Unwrapped Skills (Students need to be able to do)	Bloom's Taxonomy Levels	Webb's DOK
the Fundamental Theorem of Algebra.	Know	Understand	1
the Fundamental Theorem of Algebra.	Apply	Apply	2

the Remainder Theorem	Understand	Understand	2
it to solve problems.	Use	Analyze	3
zeros of polynomials when suitable factorizations are available,	Identify	Apply	1
the zeros to sketch the function defined by the polynomial	Use	Analyze	4
rational equations where numerators and denominators are polynomials and where extraneous solutions may result.	Solve	Analyze	3
for real and complex roots of polynomials	Solve	Analyze	2
rational functions	Explore	Analyze	4
mathematical operations on complex numbers	Perform	Apply	2

### **Essential Questions:**

1. Why is the Fundamental Theorem of Algebra important for solving polynomial equations?
2. What are the key characteristics of polynomial and rational function graphs and how are they found?
3. How do you solve polynomial and rational inequalities?

### **Enduring Understanding/Big Ideas:**

1. The Fundamental Theorem of Algebra is used to determine how many solutions a polynomial has.
2. The key characteristics of rational and polynomial function graphs can be found using various methods and include domain, range, asymptotes, intercepts, holes, end behavior, and turning points.
3. Use sign diagrams to evaluate and solve polynomial and rational inequalities.

### **Unit Vocabulary:**

Academic Cross-Curricular Words	Content/Domain Specific
	Quadratic Polynomial Rational Zeros Asymptotes Holes

### **Resources for Vocabulary Development:** Textbook

## Topic 1: Polynomial Functions

### **Engaging Experience 1**

**Title:** Finding Zeros of Polynomials

**Suggested Length of Time:** 1 class period

#### **Standards Addressed**

*Priority:*

- Alg2.NQ.B.3: Know and apply the Fundamental Theorem of Algebra.
- MW.MA.120.5: Solve for real and complex roots of polynomials
- Alg2.APR.A.2: Understand the Remainder Theorem and use it to solve problems.
- Alg2.APR.A.5: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to sketch the function defined by the polynomial.

*Supporting:*

- MW.MA.120.3: Perform mathematical operations on complex numbers.
- ISTE-EMPOWERED LEARNER1.C - use technology to seek feedback that informs and improves their practice and to demonstrate their learning in a variety of ways.

**Detailed Description/Instructions:** Students will be given a variety of polynomial functions and asked to use technology to determine potential zeros of the function. They will then use synthetic or polynomial division to reduce the function and identify zeros. Some zeros will be complex.

**Bloom's Levels:** Analyze

**Webb's DOK:** 3

### **Engaging Experience 2**

**Title:** Exploring Graphs of Polynomials with a Graphing Calculator

**Suggested Length of Time:** ½ of a class period

#### **Standards Addressed**

*Priority:*

- Alg2.APR.A.5: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to sketch the function defined by the polynomial.
- MW.MA.120.5: Solve for real and complex roots of polynomials.

*Supporting:*

- ISTE-COMPUTATIONAL THINKER.5.C - break problems into component parts, extract key information, and develop descriptive models to understand complex systems or facilitate problem-solving.

**Detailed Description/Instructions:** Students will explore the graphs of polynomials with a graphing calculator. They will experiment with the effect of the degree on the end behavior. They will experiment with the multiplicities of zeros and complex zeros. Students will generalize their observations to graph without a calculator.

**Bloom's Levels:** Analyze

**Webb's DOK:** 4

### **Engaging Experience 3**

**Title:** Polynomial Inequalities with White Boards

**Suggested Length of Time:** 30 minutes

**Standards Addressed**

*Priority:*

- MW.MA.120.5: Solve for real and complex roots of polynomials.

**Detailed Description/Instructions:** Students will find the zeros of a polynomial and use them to create a sign chart to solve inequalities.

**Bloom's Levels:** Analyze

**Webb's DOK:** 2

## Topic 2: Rational Functions

### **Engaging Experience 1**

**Title:** Analyzing Rational Functions

**Suggested Length of Time:** 2 class periods

#### **Standards Addressed**

*Priority:*

- GA.MM4A1. Students will explore rational functions.
  - Investigate and explain characteristics of rational functions, including domain, range, zeros, points of discontinuity, intervals of increase and decrease, rates of change, local and absolute extrema, symmetry, asymptotes, and end behavior.
  - Find inverses of rational functions, discussing domain and range, symmetry, and function composition.
  - Solve rational equations and inequalities analytically, graphically, and by using appropriate technology.
- Alg2.REI.A.2: Solve rational equations where numerators and denominators are polynomials and where extraneous solutions may result.

*Supporting:*

- ISTE-COMPUTATIONAL THINKER.5.C - break problems into component parts, extract key information, and develop descriptive models to understand complex systems or facilitate problem-solving.

**Detailed Description/Instructions:** Students will be tasked with taking a variety of rational functions and analyzing them to find domain, range, holes, asymptotes, end behavior, intercepts, extrema, intervals of increase and decrease. They will then create graphs of these functions both by hand and with the assistance of technology.

**Bloom's Levels:** Analyze

**Webb's DOK:** 4

### **Engaging Experience 2**

**Title:** Rational Inequalities with White Boards

**Suggested Length of Time:** 30 minutes

#### **Standards Addressed**

*Priority:*

- Solve rational equations where numerators and denominators are polynomials and where extraneous solutions may result.



**Detailed Description/Instructions:** Students will find the zeros of a rational function and the holes. They will use them to create a sign chart to solve inequalities.

**Bloom's Levels:** Analyze

**Webb's DOK:** 4

## Engaging Scenario

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will be broken into groups and given sheets of paper of various sizes. They will construct an open topped box of maximum volume. Their product will be a physical box, the equation modeling their boxes volume, a justification of their findings.

## Summary of Engaging Learning Experiences for Topics

Topic	Engaging Experience Title	Description	Suggested Length of Time
Polynomial Functions	Finding Zeros of Polynomials	Students will be given a variety of polynomial functions and asked to use technology to determine potential zeros of the function. They will then use synthetic or polynomial division to reduce the function and identify zeros. Some zeros will be complex.	1 class period
Polynomial Functions	Exploring Graphs of Polynomials with a Graphing Calculator	Students will explore the graphs of polynomials with a graphing calculator. They will experiment with the effect of the degree on the end behavior. They will experiment with the multiplicities of zeros and complex zeros. Students will generalize their observations to graph without a calculator.	$\frac{1}{2}$ of a class period
Polynomial Functions	Polynomial Inequalities with white boards	Students will find the zeros of a polynomial and use them to create a sign chart to solve inequalities.	30 minutes
Rational Functions	Analyzing Rational Functions	Students will be tasked with taking a variety of rational functions and analyzing them to find domain, range, holes, asymptotes, end behavior, intercepts, extrema, intervals of increase and decrease. They will then create graphs of these functions both by hand and with the assistance of technology.	2 class periods
Rational Functions	Rational Inequalities with White Boards	Students will find the zeros of a rational function and the holes. They will use them to create a sign chart to solve inequalities.	30 minutes

## Unit 3: Exponential and Logarithmic Functions

**Subject:** Math Analysis

**Grade:** 10, 11, 12

**Name of Unit:** Exponential and Logarithmic Functions

**Length of Unit:** 12 class periods

**Overview of Unit:** In this unit students will work with a variety of concepts and skills related to exponential and logarithmic functions. They will graph both types of functions and transform them. They will solve a variety equation formats. They will work with real world scenarios involving exponential and logarithmic functions.

### Priority Standards for unit:

- Alg2.SSE.A.1: Develop the definition of logarithms based on properties of exponents.
- Alg2.SSE.A.2: Use the inverse relationship between exponents and logarithms to solve exponential and logarithmic equations.
- Alg2.SSE.A.3: Use properties of logarithms to solve equations or find equivalent expressions.
- Alg2.BF.A.3: Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for linear, quadratic, cubic, square and cube root, absolute value, exponential and logarithmic functions.
- MW.MA.120.7: Solve exponential and logarithmic equations.

### Supporting Standards for unit:

- ISTE-KNOWLEDGE COLLECTOR.3. A - plan and employ effective research strategies to locate information and other resources for their intellectual or creative pursuits.
- ISTE-KNOWLEDGE COLLECTOR.3. B - evaluate the accuracy, perspective, credibility and relevance of information, media, data or other resources.
- ISTE-COMPUTATIONAL THINKER.5. A - formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.
- ISTE-COMPUTATIONAL THINKER.5. B - collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.
- ISTE-CREATIVE COMMUNICATOR.6. D - publish or present content that customizes the message and medium for their intended audiences.
- ISTE-EMPOWERED LEARNER1.C - use technology to seek feedback that informs and improves their practice and to demonstrate their learning in a variety of ways.

- ISTE-KNOWLEDGE COLLECTOR.3.D - build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.

<b>Unwrapped Concepts (Students need to know)</b>	<b>Unwrapped Skills (Students need to be able to do)</b>	<b>Bloom's Taxonomy Levels</b>	<b>Webb's DOK</b>
the definition of logarithms based on properties of exponents.	Develop	Apply	2
Use the inverse relationship between exponents and logarithms to <u>solve</u> exponential and logarithmic equations.	Solve	Apply	2
Use properties of logarithms to solve equations or <u>find</u> equivalent expressions.	Find	Apply	2
the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for linear, quadratic, cubic, square and cube root, absolute value, exponential and logarithmic functions.	Describe	Analyze	2
exponential and logarithmic equations	Solve	Apply	3

### **Essential Questions:**

1. How are the graphs of exponential functions related to graphs of logarithmic functions?
2. Why do the properties of logarithms parallel the properties of exponents?
3. How are exponential and logarithmic equations solved?
4. How are exponential and logarithmic functions applied in the real world?

### **Enduring Understanding/Big Ideas:**

1. Logarithmic functions are inverses of exponential functions. Their graphs are reflections over the line  $y=x$ .
2. Logarithms are defined by an exponential statement and the properties of logarithms are developed by using the properties of exponents.
3. Exponential and logarithmic equations are solved by using the properties to rewrite the equation into a simpler form. Logarithmic equations are often rewritten as exponential equations.
4. Exponential functions are used to model growth or decay, such as population. Logarithmic scales are often used in science, such as loudness of sound.

**Unit Vocabulary:**

Academic Cross-Curricular Words	Content/Domain Specific
	Exponential Function Logarithmic Function Natural Logarithm

**Resources for Vocabulary Development:** textbook

## Topic 1: Graphing

### **Engaging Experience 1**

**Title:** Graphing Exponential and Logarithmic Functions

**Suggested Length of Time:** 1 class period

#### **Standards Addressed**

*Priority:*

- Alg2.BF.A.3: Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for linear, quadratic, cubic, square and cube root, absolute value, exponential and logarithmic functions.

**Detailed Description/Instructions:** Students will be given an exponential or logarithmic function and tasked with graphing the function,  $y = x$ , and the inverse function on the same set of axes. Students will then identify the domain and range of each function.

**Bloom's Levels:** Analyze

**Webb's DOK:** 2

## Topic 2: Logs

### **Engaging Experience 1**

**Title:** Definition and Properties of Logarithms with White Boards

**Suggested Length of Time:** 1 class period

**Standards Addressed**

*Priority:*

- Develop the definition of logarithms based on properties of exponents.

**Detailed Description/Instructions:** Students will use the definition of logarithms to evaluate expression. Students will use the properties of logarithms to expand and condense expressions.

**Bloom's Levels:** Apply

**Webb's DOK:** 2



## Topic 3: Solving Equations

### **Engaging Experience 1**

**Title:** Solving Exponential and Logarithmic Equations Using a Variety of Methods

**Suggested Length of Time:** 2 class periods

### **Standards Addressed**

#### *Priority:*

- Alg2.SSE.A.2: Use the inverse relationship between exponents and logarithms to solve exponential and logarithmic equations.
- Alg2.SSE.A.3: Use properties of logarithms to solve equations or find equivalent expressions.
- MW.MA.120.7: Solve exponential and logarithmic equations.

#### *Supporting:*

- ISTE-EMPOWERED LEARNER1.C - use technology to seek feedback that informs and improves their practice and to demonstrate their learning in a variety of ways.

**Detailed Description/Instructions:** Students will be given exponential and logarithmic functions in a variety of formats. They will be asked to use their knowledge of the inverse relationship of the functions and exponentials and logarithmic properties to solve those equations. They will also use technology to confirm solutions and examine graphical representations of those solutions.

**Bloom's Levels:** Apply

**Webb's DOK:** 3

## Topic 4: Applications

### **Engaging Experience 1**

**Title:** Exponential Growth and Decay

**Suggested Length of Time:** 1 class period

#### **Standards Addressed**

*Priority:*

- Solve exponential and logarithmic equations.

*Supporting:*

- ISTE-KNOWLEDGE COLLECTOR.3.D - build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.

**Detailed Description/Instructions:** Students will solve a variety of exponential growth applications. Population growth, radioactive decay, continuously compounded interest will be included.

**Bloom's Levels:** Apply

**Webb's DOK:** 3

## Engaging Scenario

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) The students will work in groups and select a country to research a Logistic population growth model. Students will use technology to plot the country's historic population data and develop a logistic growth function for that data. Students will produce a PowerPoint slide displaying their findings.

## Summary of Engaging Learning Experiences for Topics

Topic	Engaging Experience Title	Description	Suggested Length of Time
Graphing	Graphing Exponential and Logarithmic Functions	Students will be given an exponential or logarithmic function and tasked with graphing the function, $y = x$ , and the inverse function on the same set of axes. Students will then identify the domain and range of each function.	1 class period
Logs	Definition and Properties of Logarithms with White Boards	Students will use the definition of logarithms to evaluate expression. Students will use the properties of logarithms to expand and condense expressions.	1 class period
Solving Equations	Solving Exponential and Logarithmic Equations Using a Variety of Methods	Students will be given exponential and logarithmic functions in a variety of formats. They will be asked to use their knowledge of the inverse relationship of the functions and exponentials and logarithmic properties to solve those equations. They will also use technology to confirm solutions and examine graphical representations of those solutions.	2 class periods
Applications	Exponential Growth and Decay	Students will solve a variety of exponential growth applications. Population growth, radioactive decay, continuously compounded interest will be included.	1 class period

## Unit 4: Sequences and Series

**Subject:** Math Analysis

**Grade:** 10, 11, 12

**Name of Unit:** Sequences and Series

**Length of Unit:** 5-6 class periods

**Overview of Unit:** In this unit students will write and use formulas for a variety of sequences and series.

**Priority Standards for unit:**

- MM4A9. Students will use sequences and series.
  - a. Use and find recursive and explicit formulas for the terms of sequences.
  - b. Recognize and use simple arithmetic and geometric sequences.
  - c. Find and apply the sums of finite and, where appropriate, infinite arithmetic and geometric series.
  - d. Use summation notation to explore finite series.

**Supporting Standards for unit:**

- EK 4.1A1: The  $n$ th partial sum is defined as the sum of the first  $n$  terms of a sequence.

Unwrapped Concepts (Students need to know)	Unwrapped Skills (Students need to be able to do)	Bloom's Taxonomy Levels	Webb's DOK
and find recursive and explicit formulas for the terms of sequences.	Use	Understand	1
Find recursive and explicit formulas for the terms of sequences	Find	Analyze	3
and use simple arithmetic and geometric sequences.	Recognize	Understand	1
Recognize and simple arithmetic and geometric sequences.	Use	Apply	2
and apply the sums of finite and, where appropriate, infinite arithmetic and geometric series.	Find	Apply	2
Find and the sums of finite and, where appropriate, infinite arithmetic and geometric series.	Apply	Apply	3
summation notation to explore finite series.	Use	Analyze	3

**Essential Questions:**

1. Why can sequences be defined using multiple formulas?
2. How is an arithmetic sequence distinguished from an arbitrary sequence and how is that knowledge used?
3. How is a geometric sequence distinguished from an arbitrary sequence and how is that knowledge used?

**Enduring Understanding/Big Ideas:**

1. Sequence formulas can be written recursively (based on the previous term or terms) or explicitly (finding the  $n^{\text{th}}$  term).
2. Successive terms of an arithmetic sequence have a common difference. This provides guidance on which formulas to use to calculate terms and partial sums.
3. Successive terms of a geometric sequence have a common ratio. This provides guidance on which formulas to use to calculate terms, partial sums, and infinite sums.

**Unit Vocabulary:**

Academic Cross-Curricular Words	Content/Domain Specific
	Recursive Formula Arithmetic Sequence Geometric Sequence Series Summation or Sigma Notation Infinite Geometric Series

**Resources for Vocabulary Development:** textbook

## Topic 1: General Sequences and Series

### **Engaging Experience 1**

**Title:** Modeling Sequences and Series

**Suggested Length of Time:** 1 class period

#### **Standards Addressed**

*Priority:*

- MM4A9. Students will use sequences and series.
  - a. Use and find recursive and explicit formulas for the terms of sequences.
  - b. Recognize and use simple arithmetic and geometric sequences.
  - c. Find and apply the sums of finite and, where appropriate, infinite arithmetic and geometric series.
  - d. Use summation notation to explore finite series.

*Supporting:*

- EK 4.1A1: The  $n$ th partial sum is defined as the sum of the first  $n$  terms of a sequence.

**Detailed Description/Instructions:** Students will work with a variety of sequences of numbers and develop recursive and/or explicit formulas to represent them. They will be introduced to summation notation practice calculating partial sums of these sequences.

**Bloom's Levels:** Analyze

**Webb's DOK:** 3

## Topic 2: Arithmetic Sequences and Series

### **Engaging Experience 1**

**Title:** Identify Arithmetic Sequences and Series and Applying Formulas with White Board

**Suggested Length of Time:**  $\frac{1}{2}$  of a class period

#### **Standards Addressed**

*Priority:*

- MM4A9. Students will use sequences and series.
  - a. Use and find recursive and explicit formulas for the terms of sequences.
  - b. Recognize and use simple arithmetic and geometric sequences.
  - c. Find and apply the sums of finite and, where appropriate, infinite arithmetic and geometric series.
  - d. Use summation notation to explore finite series.

*Supporting:*

- EK 4.1A1: The  $n$ th partial sum is defined as the sum of the first  $n$  terms of a sequence.

**Detailed Description/Instructions:** Students will be given sequences and then determine the type of sequence. Students will write explicit formulas for the sequences. Students will use the formulas to find other terms of the sequences and to evaluate partial sums.

**Bloom's Levels:** Apply

**Webb's DOK:** 2



## Topic 3: Geometric Sequences and Series

### **Engaging Experience 1**

**Title:** Identify Geometric Sequences and Series and Applying Formulas with White Board

**Suggested Length of Time:**  $\frac{1}{2}$  of a class period

#### **Standards Addressed**

*Priority:*

- MM4A9. Students will use sequences and series.
  - a. Use and find recursive and explicit formulas for the terms of sequences.
  - b. Recognize and use simple arithmetic and geometric sequences.
  - c. Find and apply the sums of finite and, where appropriate, infinite arithmetic and geometric series.
  - d. Use summation notation to explore finite series

**Detailed Description/Instructions:** Students will be given sequences and then determine the type of sequence. Students will write explicit formulas for the sequences. Students will use the formulas to find other terms of the sequences and to evaluate partial sum.

**Bloom's Levels:** Apply

**Webb's DOK:** 2

## Engaging Scenario

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Student groups will be given a complex number in the form  $a + bi$  and tasked with expanding and simplifying the complex number when raised to the 5<sup>th</sup> power. They will research the binomial theorem and the sequence of its coefficients in order to accomplish this task. Students will show the expansion and accompanying simplification and verify their results using appropriate technology.

## Summary of Engaging Learning Experiences for Topics

<b>Topic</b>	<b>Engaging Experience Title</b>	<b>Description</b>	<b>Suggested Length of Time</b>
General Sequences and Series	Modeling Sequences and Series	Students will work with a variety of sequences of numbers and develop recursive and/or explicit formulas to represent them. They will be introduced to summation notation practice calculating partial sums of these sequences.	1 class period
Arithmetic Sequences and Series	Identify Arithmetic Sequences and Series and Applying Formulas with White Board	Students will be given sequences and then determine the type of sequence. Students will write explicit formulas for the sequences. Students will use the formulas to find other terms of the sequences and to evaluate partial sums.	½ of a class period
Geometric Sequences and Series	Identify Geometric Sequences and Series and Applying Formulas with White Board	Students will be given sequences and then determine the type of sequence. Students will write explicit formulas for the sequences. Students will use the formulas to find other terms of the sequences and to evaluate partial sum	½ of a class period

## Unit 5: Conics

**Subject:** Math Analysis

**Grade:** 10, 11, 12

**Name of Unit:** Conics

**Length of Unit:** 7-8 class periods

**Overview of Unit:** In this unit students will write equations and graph each type of conic section: circle, ellipse, hyperbola, parabola.

**Priority Standards for unit:**

- AL.9-12. F-CS.1.a Formulate equations of conic sections from their determining characteristics.
  - Write equations of parabolas in vertex form (PC-P.2)
  - Write equations of circles in standard form (PC-P.5)
  - Write equations of ellipses in standard form (PC-P.9)
  - Write equations of hyperbolas in standard form (PC-P.12)
  - Convert equations of conic sections from general to standard form (PC-P.13)

**Supporting Standards for unit:**

- AL.9-12. F-CS.1 Create graphs of conic sections, including parabolas, hyperbolas, ellipses, circles, and degenerate conics, from second-degree equations.

Unwrapped Concepts (Students need to know)	Unwrapped Skills (Students need to be able to do)	Bloom's Taxonomy Levels	Webb's DOK
equations of conic sections from their determining characteristics.	Formulate	Evaluate	3

**Essential Questions:**

1. How do the properties of a conic section determine its graph and equation?
2. Why are transformations important when graphing or writing an equation of a conic section?

**Enduring Understanding/Big Ideas:**

1. The geometric definition of a conic section will result in a standard equation and the properties (foci, vertices, etc.) can be found from the equation.
2. Any given conic is simply the standard conic translated and stretched and equations and graphs can easily be found when the transformation is known.

**Unit Vocabulary:**

Academic Cross-Curricular Words	Content/Domain Specific
	Conic Circle Ellipse Hyperbola Parabola Transformation Focus Vertex Directrix Asymptotes

**Resources for Vocabulary Development:** Textbook

# Topic 1: Conics

## **Engaging Experience 1**

**Title:** Conic Sections Centered about the Origin with the White Boards

**Suggested Length of Time:** 3 class periods

### **Standards Addressed**

*Priority:*

- AL.9-12.F-CS.1.a Formulate equations of conic sections from their determining characteristics.
  - Write equations of parabolas in vertex form (PC-P.2)
  - Write equations of circles in standard form (PC-P.5)
  - Write equations of ellipses in standard form (PC-P.9)
  - Write equations of hyperbolas in standard form (PC-P.12)
  - Convert equations of conic sections from general to standard form (PC-P.13)

*Supporting:*

- AL.9-12.F-CS.1 Create graphs of conic sections, including parabolas, hyperbolas, ellipses, circles, and degenerate conics, from second-degree equations.

**Detailed Description/Instructions:** Students will be given equations of conic sections in standard form and will then sketch their graphs and identify their key features. Students will be given the graph of a conic and then write its equation in standard form. When given the key features of a conic, students will write the equation and graph.

**Bloom's Levels:** Evaluate

**Webb's DOK:** 3

## **Engaging Experience 2**

**Title:** Transforming Conic Sections

**Suggested Length of Time:** 1 class period

### **Standards Addressed**

*Priority:*

- AL.9-12.F-CS.1.a Formulate equations of conic sections from their determining characteristics.
  - Write equations of parabolas in vertex form (PC-P.2)
  - Write equations of circles in standard form (PC-P.5)
  - Write equations of ellipses in standard form (PC-P.9)
  - Write equations of hyperbolas in standard form (PC-P.12)
  - Convert equations of conic sections from general to standard form (PC-P.13)

*Supporting:*

- AL.9-12.F-CS.1 Create graphs of conic sections, including parabolas, hyperbolas, ellipses, circles, and degenerate conics, from second-degree equations.

**Detailed Description/Instructions:** Students will be given translated conic sections in general and standard form and tasked with graphing those sections. If the section is in general form they will need to convert it to standard form by completing the square.

**Bloom's Levels:** Evaluate

**Webb's DOK:** 3

## Engaging Scenario

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will find an image of a conic section (ellipse, hyperbola, or parabola) in architecture. They will then import this image into Desmos or another graphing technology application and develop an equation to model their conic section image.



## Summary of Engaging Learning Experiences for Topics

Topic	Engaging Experience Title	Description	Suggested Length of Time
Conics	Conic Sections Centered about the Origin with the White Boards	Students will be given equations of conic sections in standard form and will then sketch their graphs and identify their key features. Students will be given the graph of a conic and then write its equation in standard form. When given the key features of a conic, students will write the equation and graph.	3 class periods
Conics	Transforming Conic Sections	Students will be given translated conic sections in general and standard form and tasked with graphing those sections. If the section is in general form, they will need to convert it to standard form by completing the square.	1 class period

## Unit 6: Limits

**Subject:** Math Analysis

**Grade:** 10, 11, 12

**Name of Unit:** Limits

**Length of Unit:** 10-12 class periods

**Overview of Unit:** In this unit the student will evaluate limits and use limit laws. The student will apply limits to sequences, tangent lines, and areas.

### Priority Standards for unit:

- EK 1.1A1: Given a function, the limit of as approaches is a real number if can be made arbitrarily close to by taking sufficiently close to (but not equal to). If the limit exists and is a real number, then the common notation is  $\lim$ .
- \*EK 1.2A1: A function is continuous at provided that exists,  $\lim$  exists, and  $\lim$ .
- EK 1.1A2: The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.

### Supporting Standards for unit:

- EK 1.1A3: A limit might not exist for some functions at particular values of. Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right.
- EK 1.1C1: Limits of sums, differences, products, quotients, and composite functions can be found using the basic theorems of limits and algebraic rules.
- EK 1.2A2: Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains.
- EK 1.2A3: Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.
- EK 1.1D1: Asymptotic and unbounded behavior of functions can be explained and described using limits.
- ISTE-EMPOWERED LEARNER.1.C - use technology to seek feedback that informs and improves their practice and to demonstrate their learning in a variety of ways.
- ISTE-INNOVATIVE DESIGNER.4.A - know and use a deliberate design process for generating ideas, testing theories, creating innovative artifacts or solving authentic problems.
- ISTE-COMPUTATIONAL THINKER.5.A - formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.

<b>Unwrapped Concepts (Students need to know)</b>	<b>Unwrapped Skills (Students need to be able to do)</b>	<b>Bloom's Taxonomy Levels</b>	<b>Webb's DOK</b>
Given a function, the limit of as approaches is a real number if can be made arbitrarily close to by taking sufficiently close to (but not equal to). If the limit exists and is a real number, then the common notation is $\lim$		Apply	4
A function is continuous at provided that exists, $\lim$ exists, and $\lim$		Analyze	2
The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.		Analyze	3

**Essential Questions:**

1. How can limits be used to examine a function?
2. How can you find the slope of a tangent line at any point on a curve?
3. How can you find the area under a curve?

**Enduring Understanding/Big Ideas:**

1. Limits can be used to determine the limit at a given domain value, continuity, end behavior, and asymptotic behavior.
2. Applying limits to the slope formula will result in finding the slope of a tangent line to a curve at a given point -- also known as the derivative.
3. By drawing increasing numbers of rectangles beneath a curve the area under that curve can be approximated.

**Unit Vocabulary:**

<b>Academic Cross-Curricular Words</b>	<b>Content/Domain Specific</b>
	Limit Continuous One-sided Limit Derivative Integral

**Resources for Vocabulary Development:** Textbook

# Topic 1: Limits

## **Engaging Experience 1**

**Title:** Evaluating Limits Graphically, Numerically, Algebraically

**Suggested Length of Time:** 2 class periods

### **Standards Addressed**

*Priority:*

- EK 1.1A1: Given a function, the limit of as approaches is a real number if can be made arbitrarily close to by taking sufficiently close to (but not equal to). If the limit exists and is a real number, then the common notation is  $\lim$ .
- \*EK 1.2A1: A function is continuous at provided that exists,  $\lim$  exists, and  $\lim$ .

*Supporting:*

- EK 1.1C1: Limits of sums, differences, products, quotients, and composite functions can be found using the basic theorems of limits and algebraic rules.
- EK 1.1A2: The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.

**Detailed Description/Instructions:** The student will find the limit of functions given in a variety of forms. They will use graphing calculators to find limits numerically and graphically. The student will apply the limit laws to find limits algebraically.

**Bloom's Levels:** Analyze

**Webb's DOK:** 4

## **Engaging Experience 2**

**Title:** Investigating One-sided and Infinite Limits

**Suggested Length of Time:** 2 class periods

### **Standards Addressed**

*Priority:*

- EK 1.1A2: The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.
- \*EK 1.2A1: A function is continuous at provided that exists,  $\lim$  exists, and  $\lim$ .

*Supporting:*

- EK 1.1A3: A limit might not exist for some functions at particular values of. Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right.
- EK 1.2A3: Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.
- EK 1.1D1: Asymptotic and unbounded behavior of functions can be explained and described using limits.

**Detailed Description/Instructions:** Students will use one-sided limits to investigate the behavior of various functions at specific values and discontinuities. Students will also use infinite limits to assess the end behavior of various functions.

**Bloom's Levels:** Analyze

**Webb's DOK:** 3

## Topic 2: Applications

### **Engaging Experience 1**

**Title:** Maximizing and Minimizing

**Suggested Length of Time:** 2 class periods

**Standards Addressed**

*Priority:*

- EK 1.1A1: Given a function, the limit of as approaches is a real number if can be made arbitrarily close to by taking sufficiently close to (but not equal to). If the limit exists and is a real number, then the common notation is  $\lim$ .

*Supporting:*

- ISTE-INNOVATIVE DESIGNER.4.A - know and use a deliberate design process for generating ideas, testing theories, creating innovative artifacts or solving authentic problems.

**Detailed Description/Instructions:** Students will be given real world scenarios that involve maximizing or minimizing a function. Students will then have to use limits and derivatives to evaluate the functions and determine those maximizing or minimizing values.

**Bloom's Levels:** Apply

**Webb's DOK:** 4

### **Engaging Experience 2**

**Title:** Find the Area Under a Curve

**Suggested Length of Time:** 2 class periods

**Standards Addressed**

*Priority:*

- EK 1.1A1: Given a function, the limit of as approaches is a real number if can be made arbitrarily close to by taking sufficiently close to (but not equal to). If the limit exists and is a real number, then the common notation is  $\lim$ .

**Detailed Description/Instructions:** Students will be given various functions and asked to approximate the area under the curve using left-hand rectangles and right-hand rectangles. They will make conjectures about if their areas are overestimates or underestimates. They will also make conjectures about what methods could be used to make their approximations more accurate.

**Bloom's Levels:** Apply

**Webb's DOK:** 4

## Engaging Scenario

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will be broken into groups and assigned a function. They will be tasked with creating a spreadsheet that can be used to calculate the area of rectangles under the function in order to approximate the total area. They will then apply the concepts of limits to project the exact area contained under the curve.

## Summary of Engaging Learning Experiences for Topics

Topic	Engaging Experience Title	Description	Suggested Length of Time
Limits	Evaluating Limits Graphically, Numerically, Algebraically	The student will find the limit of functions given in a variety of forms. They will use graphing calculators to find limits numerically and graphically. The student will apply the limit laws to find limits algebraically.	2 class periods
Limits	Investigating One-sided and Infinite Limits	Students will use one-sided limits to investigate the behavior of various functions at specific values and discontinuities. Students will also use infinite limits to assess the end behavior of various functions.	2 class periods
Applications	Maximizing and Minimizing	Students will be given real world scenarios that involve maximizing or minimizing a function. Students will then have to use limits and derivatives to evaluate the functions and determine those maximizing or minimizing values.	2 class periods
Applications	Find the Area Under a Curve	Students will be given various functions and asked to approximate the area under the curve using left-hand rectangles and right-hand rectangles. They will make conjectures about if their areas are overestimates or underestimates. They will also make conjectures about what methods could be used to make their approximations more accurate.	2 class periods



## Unit 7: Systems

**Subject:** Math Analysis

**Grade:** 10, 11, 12

**Name of Unit:** Systems

**Length of Unit:** 7-8 class periods

**Overview of Unit:** In this unit students will solve systems of nonlinear equations and inequalities. Students will also learn how to decompose a rational function into partial fractions.

**Priority Standards for unit:**

- Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.
- DESE.PostAlg2.7: Perform partial fraction decomposition of rational functions

**Supporting Standards for unit:**

- ISTE-COMPUTATIONAL THINKER.5.C - break problems into component parts, extract key information, and develop descriptive models to understand complex systems or facilitate problem-solving.

Unwrapped Concepts (Students need to know)	Unwrapped Skills (Students need to be able to do)	Bloom's Taxonomy Levels	Webb's DOK
systems of equations that may include non-linear equations and inequalities.	Create	Apply	2
systems of equations that may include non-linear equations and inequalities.	Solve	Apply	2
partial fraction decomposition of rational functions	Perform	Analyze	4

**Essential Questions:**

- Why are partial fractions useful?
- How can systems of linear and nonlinear equations and inequalities be solved?

**Enduring Understanding/Big Ideas:**

- Rational functions can be broken down into the sum of simpler fractions with linear and quadratic denominators.
- Methods for solving systems of equations include substitution, elimination, graphing. Systems of inequalities can be solved by graphing and shading the feasible region.

**Unit Vocabulary:**

Academic Cross-Curricular Words	Content/Domain Specific
	Partial fractions Nonlinear

**Resources for Vocabulary Development:** Textbook

## Topic 1: Systems

### **Engaging Experience 1**

**Title:** Extending Methods of Solving Linear Systems to Nonlinear on White Boards

**Suggested Length of Time:**  $\frac{1}{2}$  of a class period

**Standards Addressed**

*Priority:*

- Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.

**Detailed Description/Instructions:** The student will apply substitution, elimination, and graphing to systems involving nonlinear equations.

**Bloom's Levels:** Apply

**Webb's DOK:** 2

### **Engaging Experience 2**

**Title:** Graphically Solving Systems of Inequalities and Applying to Real World Situations

**Suggested Length of Time:** 1 class period

**Standards Addressed**

*Priority:*

- Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.

**Detailed Description/Instructions:** Students will be presented with systems of inequalities involving linear and nonlinear inequalities and then use the process in linear programming.

**Bloom's Levels:** Apply

**Webb's DOK:** 2

## Topic 2: Partial Fractions

### **Engaging Experience 1**

**Title:** Partial Fractions

**Suggested Length of Time:** 1 class period

**Standards Addressed**

*Priority:*

- DESE.PostAlg2.7: Perform partial fraction decomposition of rational functions

**Detailed Description/Instructions:** Students will decompose rational functions into partial fractions involving linear, repeating, and quadratic denominators.

**Bloom's Levels:** Analyze

**Webb's DOK:** 4

## Engaging Scenario

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will be broken into groups and assigned a linear programming scenario. They will then be tasked with writing the inequalities described, constructing a graph, finding vertices, and determining the maximum or minimum of the scenario.

## Summary of Engaging Learning Experiences for Topics

Topic	Engaging Experience Title	Description	Suggested Length of Time
Systems	Extending Methods of solving Linear Systems to Nonlinear on White Boards	The student will apply substitution, elimination, and graphing to systems involving nonlinear equations.	½ of a class period
Systems	Graphically Solving Systems of Inequalities and Applying to Real World Situations	Students will be presented with systems of inequalities involving linear and nonlinear inequalities and then use the process in linear programming.	1 class period
Partial Fractions	Partial Fractions	Students will decompose rational functions into partial fractions involving linear, repeating, and quadratic denominators.	1 class period

## Unit 8: Trigonometry

**Subject:** Math Analysis

**Grade:** 10, 11, 12

**Name of Unit:** Trigonometry

**Length of Unit:** 7-10 class periods

**Overview of Unit:** In this unit students will solve a variety of trigonometric equations as well as graph polar and parametric equations.

**Priority Standards for unit:**

- DESE.PostAlg2.3: Solve equations involving trigonometric functions.
- DESE.PostAlg2.6: Graph using polar coordinates.
- IND.PC.PE.1: Convert between a pair of parametric equations and an equation in  $x$  and  $y$ . Model and solve problems using parametric equations.

**Supporting Standards for unit:**

- IND.PC.PE.2: Analyze planar curves, including those given in parametric form.
- ISTE-EMPOWERED LEARNER1.C - use technology to seek feedback that informs and improves their practice and to demonstrate their learning in a variety of ways.
- ISTE-KNOWLEDGE COLLECTOR.3.D - build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.

Unwrapped Concepts (Students need to know)	Unwrapped Skills (Students need to be able to do)	Bloom's Taxonomy Levels	Webb's DOK
equations involving trigonometric functions	Solve	Apply	2
using polar coordinates	Graph	Apply	2
between a pair of parametric equations and an equation in $x$ and $y$ .	Convert	Apply	1
problems using parametric equations	Model	Analyze	3
problems using parametric equations	Solve	Apply	3

**Essential Questions:**

1. How are trigonometric equations solved?
2. Why is the polar plane useful?
3. How are plane curves and parametric equations similar and how are they different?

**Enduring Understanding/Big Ideas:**

1. Trigonometric equations are solved by applying identities if necessary and then identifying the equation as linear, quadratic or another standard equation. The unit circle or an approximation is then used.
2. The polar plane allows complex graphs to be represented by a simple polar equation that will be in function form.
3. Plane curves and parametric equations both generate ordered pairs and can both be represented on the rectangular plane. The graph of parametric equations has direction. They are often used to represent the path of a particle measured in terms of time.

**Unit Vocabulary:**

Academic Cross-Curricular Words	Content/Domain Specific
	Polar Coordinates Polar Equation Complex Plane Parametric Equation Plane Curve Parameter

**Resources for Vocabulary Development:** textbook



## Topic 1: Solving Trigonometric Equations

### **Engaging Experience 1**

**Title:** Solving Trig Equations

**Suggested Length of Time:** 2 class periods

**Standards Addressed**

*Priority:*

- DESE.PostAlg2.3: Solve equations involving trigonometric functions.

**Detailed Description/Instructions:** Students will be given sets of trigonometric equations in various formats. They will be tasked with solving them using methods like algebraic manipulation, factoring, and double angle substitution. Students will have to check the validity of their answers using algebraic methods and technology.

**Bloom's Levels:** Apply

**Webb's DOK:** 2

## Topic 2: Polar Coordinates and Graphing

### **Engaging Experience 1**

**Title:** Polar Graphing

**Suggested Length of Time:** 2 class periods

#### **Standards Addressed**

*Priority:*

- DESE.PostAlg2.6: Graph using polar coordinates.

*Supporting:*

- ISTE-EMPOWERED LEARNER1.C - use technology to seek feedback that informs and improves their practice and to demonstrate their learning in a variety of ways.

**Detailed Description/Instructions:** Students will be given a variety of equations in polar format. They will be asked to graph these functions by hand and using technology.

**Bloom's Levels:** Apply

**Webb's DOK:** 2

## Topic 3: Parametric Equations

### **Engaging Experience 1**

**Title:** Parametric Coordinates and Graphing

**Suggested Length of Time:** 2 class periods

#### **Standards Addressed**

*Priority:*

- IND.PC.PE.1: Convert between a pair of parametric equations and an equation in  $x$  and  $y$ . Model and solve problems using parametric equations.

*Supporting:*

- IND.PC.PE.2: Analyze planar curves, including those given in parametric form.

**Detailed Description/Instructions:** Students will be given sets of coordinates in rectangular and parametric format and asked to convert between the two. Students will also be given parametric equations and asked to graph them indicating direction.

**Bloom's Levels:** Analyze

**Webb's DOK:** 3

## Engaging Scenario

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will be broken into groups and given the path of two or more moving objects. Students will use parametric equations to model the movement of each object and determine will collide or pass by harmlessly.

## Summary of Engaging Learning Experiences for Topics

Topic	Engaging Experience Title	Description	Suggested Length of Time
Solving Trigonometric Equations	Solving Trig Equations	Students will be given sets of trigonometric equations in various formats. They will be tasked with solving them using methods like algebraic manipulation, factoring, and double angle substitution. Students will have to check the validity of their answers using algebraic methods and technology.	2 class periods
Polar Coordinates and Graphing	Polar Graphing	Students will be given a variety of equations in polar format. They will be asked to graph these functions by hand and using technology.	2 class periods
Parametric Equations	Parametric Coordinates and Graphing	Students will be given sets of coordinates in rectangular and parametric format and asked to convert between the two. Students will also be given parametric equations and asked to graph them indicating direction.	2 class periods

# Unit of Study Terminology

**Appendices:** All Appendices and supporting material can be found in this course's shell course in the District's Learning Management System.

**Assessment Leveling Guide:** A tool to use when writing assessments in order to maintain the appropriate level of rigor that matches the standard.

**Big Ideas/Enduring Understandings:** Foundational understandings teachers want students to be able to discover and state in their own words by the end of the unit of study. These are answers to the essential questions.

**Engaging Experience:** Each topic is broken into a list of engaging experiences for students. These experiences are aligned to priority and supporting standards, thus stating what students should be able to do. An example of an engaging experience is provided in the description, but a teacher has the autonomy to substitute one of their own that aligns to the level of rigor stated in the standards.

**Engaging Scenario:** This is a culminating activity in which students are given a role, situation, challenge, audience, and a product or performance is specified. Each unit contains an example of an engaging scenario, but a teacher has the ability to substitute with the same intent in mind.

**Essential Questions:** Engaging, open-ended questions that teachers can use to engage students in the learning.

**Priority Standards:** What every student should know and be able to do. These were chosen because of their necessity for success in the next course, the state assessment, and life.

**Supporting Standards:** Additional standards that support the learning within the unit.

**Topic:** These are the main teaching points for the unit. Units can have anywhere from one topic to many, depending on the depth of the unit.

**Unit of Study:** Series of learning experiences/related assessments based on designated priority standards and related supporting standards.

**Unit Vocabulary:** Words students will encounter within the unit that are essential to understanding. Academic Cross-Curricular words (also called Tier 2 words) are those that can be found in multiple content areas, not just this one. Content/Domain Specific vocabulary words are those found specifically within the content.