

# Park Hill School District 

Building Successful Futures • Each Student • Every Day

## High School College Algebra Curriculum

Course Description: The class consists of two Metropolitan Community College courses: Math 110, Intermediate Algebra and Math 120, College Algebra for 6 hours college credit. The catalog description for Math 110 is a study of functions and their graphs, systems of linear equations, application problems, linear and quadratic inequalities, absolute value equations and inequalities, rational exponents, radicals, quadratic equations, ratios and proportions. The catalog description for Math 120 is a study of various types of equations and inequalities, functions and their inverses, theory of higher degree equations, systems of equations, determinants, logarithmic and exponential functions, conic sections, sequences and series, and the Binomial Theorem.

Scope and Sequence:

| Timeframe | Unit | Instructional Topics |
| :---: | :---: | :--- |
| 8 class periods | 1 Variable Equations and <br> Inequalities | Topic 1: Set Theory <br> Topic 2: Properties of Real Numbers <br> Topic 3: Linear Equations <br> Topic 4: Inequalities |
| 4 class periods | 2 Variable Equations and <br> Inequalities | Topic 1: Coordinate Geometry <br> Topic 2: Linear Functions <br> Topic 3: Functions |
| 3 class periods | Systems | Topic 1: Solve and Apply |
| 5 class periods | Exponents and <br> Polynomials | Topic 1: Simplify Polynomials <br> Topic 2: Factoring |
| 6 class periods | Rationals | Topic 1: Simplify Rational Expressions <br> Topic 2: Solve and Apply |


| 6 class periods | Radicals | Topic 1: Simplifying Radicals <br> Topic 2: Solving Radical Equations <br> Topic 3: Complex Numbers |
| :---: | :---: | :--- |
| 6 class periods | Quadratics | Topic 1: Polynomial Functions <br> Topic 2: Solving and Applying |
| 9 class periods | Functions | Topic 1: Functions and Graphs <br> Topic 2: Analyzing Functions <br> Topic 3: Combining Functions <br> Topic 4: Inverse Functions |
| 10 class periods | Polynomials and <br> Rationals | Topic 1: Polynomial Functions <br> Topic 2: Rational Functions |
| 9 class periods | Exponentials and <br> Logarithms | Topic 1: Exponential Functions <br> Topic 2: Logarithmic Functions <br> Topic 3: Solve and Apply |
| 10 class periods | Conics and Systems of <br> Equations | Topic 1: Conic Sections <br> Topic 2: Systems of Equations |

## Unit 1: 1 Variable Equations and Inequalities

Subject: College Algebra
Grade: 10, 11, 12
Name of Unit: 1 Variable Equations and Inequalities
Length of Unit: 8 class periods
Overview of Unit: In this unit, students will solve and model with linear equations and inequalities, utilize the definitions of real numbers and their properties, and properly communicate solutions using the concepts of set theory.

## Priority Standards for unit:

- Alg2.REI.A.1: Create and solve equations and inequalities, including those that involve absolute value.


## Supporting Standards for unit:

- ISTE-KNOWLEDGE COLLECTOR.3.D - build knowledge by actively exploring realworld issues and problems, developing ideas and theories and pursuing answers and solutions.

| Unwrapped Concepts <br> (Students need to know) | Unwrapped Skills <br> (Students need to be able to do) | Bloom's <br> Taxonomy Levels | Webb's <br> DOK |
| :---: | :---: | :---: | :---: |
| equations and inequalities, <br> including those that involve <br> absolute value. | Create | Create | 3 |
| equations and inequalities, <br> including those that involve <br> absolute value. | Solve | Apply | 2 |

## Essential Questions:

1. How do you use interval notation to communicate solutions?
2. How do you use the properties of real numbers to evaluate expressions and solve equations?
3. How is order of operations used to isolate a variable in an equation?
4. How do you solve compound inequalities?

## Enduring Understanding/Big Ideas:

1. Write solutions from the smallest endpoint to largest endpoint using parentheses or brackets correctly.
2. They are used to make computations easier and quicker than relying only on following the order of operations.
3. Variables are isolated by applying the order of operations backwards with inverses.
4. They are solved by determining if we have an and/or statement and communicating the solution using correct notation.

Unit Vocabulary:

| Academic Cross-Curricular Words | Content/Domain Specific |
| :---: | :---: |
| Evaluate Intersection Union | Inequality <br> Absolute value <br> Algebraic expression <br> Numeric Expression <br> Additive inverse <br> Multiplicative Inverse <br> Irrational <br> Rational <br> Integer <br> Whole <br> Natural <br> Expression vs. Equation <br> Interval Notation <br> Sets <br> Element <br> Subset |

Resources for Vocabulary Development: textbook

## Topic 1: Set Theory

## Engaging Experience 1

Title: Set Activity
Suggested Length of Time: 10 minutes
Standards Addressed
Priority:

- Alg2.REI.A.1: Create and solve equations and inequalities, including those that involve absolute value.
Detailed Description/Instructions: Teacher will call out common traits among students in the class to make a set of students in the class. Based on what instructions are called out, students will stand or sit, demonstrating concepts of subsets, elements, unions, and intersections.
Bloom's Levels: Create and Apply
Webb's DOK: 3, 2


## Topic 2: Properties of Real Numbers

## Engaging Experience 1

Title: Properties with Algebra Tiles
Suggested Length of Time: 15 minutes
Standards Addressed
Priority:

- Alg2.REI.A.1: Create and solve equations and inequalities, including those that involve absolute value.
Detailed Description/Instructions: As this is the first time that students will have seen the distributive property using variables in College Algebra, we will distribute and divide variable expressions using algebra tiles. This will strengthen various properties and provide a lead-in to future lessons.
Bloom's Levels: Create and Apply
Webb's DOK: 3, 2


## Topic 3: Linear Equations

## Engaging Experience 1

Title: Reverse Procedures to demonstrate isolation
Suggested Length of Time: 10 minutes
Standards Addressed
Priority:

- Alg2.REI.A.1: Create and solve equations and inequalities, including those that involve absolute value.
Detailed Description/Instructions: Students will pair up, one will write out a procedure (i.e., the process of putting a gas nozzle in a car), the other student will write the reverse procedure (i.e., taking the nozzle out). They will then apply this to a linear equation using the reverse order of operations.
Bloom's Levels: Create and Apply
Webb's DOK: 3, 2


## Topic 4: Inequalities

## Engaging Experience 1

Title: Inequalities from a graphical/numerical perspective
Suggested Length of Time: 10 minutes

## Standards Addressed

Priority:

- Alg2.REI.A.1: Create and solve equations and inequalities, including those that involve absolute value.
Detailed Description/Instructions: Students will be given various problems that provide a more meaningful approach to solving inequalities. These examples will include types that have no solution, encompass all real numbers, solutions that contain a set and one of its subsets, and other "non-traditional" problems that lead to students thinking critically about the definitions of inequalities, and statements, and or statements.
Bloom's Levels: Create and Apply
Webb's DOK: 3, 2


## Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Mix It Up - In this activity, students will work with mixtures of two colors of beads to understand the effect of combining two different mixtures to predict the percent concentration of the final mixture.

## Summary of Engaging Learning Experiences for Topics

| Topic | Engaging <br> Experience Title | Description | Suggested <br> Length of <br> Time |
| :---: | :---: | :---: | :---: |
| Set Theory | Set Activity | Teacher will call out common traits <br> among students in the class to make a set <br> of students in the class. Based on what <br> instructions are called out, students will <br> stand or sit, demonstrating concepts of <br> subsets, elements, unions, and <br> intersections. | 10 minutes |
| Properties | Properties With <br> Algebra Tiles | As this is the first time that students will <br> have seen the distributive property using <br> variables in College Algebra, we will | 15 minutes |
| Numbers | distribute and divide variable expressions <br> using algebra tiles. This will strengthen <br> various properties and provide a lead-in <br> to future lessons. |  |  |
| Linear | Reverse Procedures <br> to demonstrate <br> isolation | Students will pair up, one will write out a <br> procedure (i.e., the process of putting a <br> gas nozzle in a car), the other student will <br> write the reverse procedure (i.e., taking <br> the nozzle out). They will then apply this <br> to a linear equation using the reverse <br> order of operations. | 10 minutes |
| Inequalities | Inequalities from a <br> graphical/numerical <br> perspective | Students will be given various problems <br> that provide a more meaningful approach <br> to solving inequalities. These examples <br> will include types that have no solution, <br> encompass all real numbers, solutions <br> that contain a set and one of its subsets, <br> and other "non-traditional" problems that <br> lead to students thinking critically about <br> the definitions of inequalities, and <br> statements, and or statements. | 10 minutes |

## Unit 2: 2 Variable Equations and Inequalities

Subject: College Algebra
Grade: 10, 11, 12
Name of Unit: 2 Variable Equations and Inequalities
Length of Unit: 4 class periods
Overview of Unit: In this unit students will graph and write linear equations. They will work with equations in standard form, point-slope form, and slope-intercept form. In addition, they will find equations that are parallel and perpendicular.

## Priority Standards for unit:

- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.


## Supporting Standards for unit:

- Alg2.IF.A.2: Translate between equivalent forms of functions.
- NMP.FF. 2 Use multiple representations of functions to interpret and describe how two quantities change together.
- NMP.FF. 3 Measure, compute, describe, and interpret rates of change of quantities embedded in multiple representations.
- ISTE-COMPUTATIONAL THINKER.5.B - collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problemsolving and decision-making.

| Unwrapped Concepts (Students <br> need to know) | Unwrapped Skills <br> (Students need to be able to do) | Bloom's <br> Taxonomy <br> Levels | Webb' <br> s DOK |
| :---: | :---: | :---: | :---: |
| key characteristics of functions <br> represented graphically, with <br> tables and with algebraic <br> symbolism to solve problems |  |  |  |
| key characteristics of functions <br> represented graphically, with <br> tables and with algebraic | Identify | Remember | 1 |
| symbolism to solve problems | Interpret |  |  |

## Essential Questions:

1. How is the slope of two lines related to their graphs?
2. How do we use given information to write the equation or construct a graph of a linear equation?
3. How is the graph of a linear equation different than the graph of a linear inequality?
4. How are relations and functions similar and different?

## Enduring Understanding/Big Ideas:

1. Find the slope of parallel and perpendicular lines and understand the relationship between the two.
2. Write and graph linear equations in Point-Slope, Standard, and Slope-Intercept forms
3. Linear equations contain the set of all solutions to an equation while linear inequalities contain the set of all solutions that satisfy an inequality.
4. Functions are relations that have a unique $y$-value for each $x$-value.

Unit Vocabulary:

| Academic Cross-Curricular Words | Content/Domain Specific |
| :---: | :---: |
| Variable | Linear |
| Slope | Coordinate plane |
| Intercept | Parallel |
| Rate of Change | Perpendicular |
| Scatterplot | Point-Slope Form |
| Intersection | Slope-Intercept Form |
| Equation | Standard Form |
| Independent Variable | Inequality |
| Dependent Variable | Function |
|  | Relation |
|  | Reciprocal |

Resources for Vocabulary Development: textbook

## Topic 1: Coordinate Geometry

## Engaging Experience 1

Title: Deriving formulas for distance and midpoint using the Pythagorean Theorem
Suggested Length of Time: 15 minutes
Standards Addressed
Priority:

- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.
Supporting:
- Alg2.IF.A.2: Translate between equivalent forms of functions.

Detailed Description/Instructions: Instead of giving students the formulas they will need for distance and midpoint, we will prove them using the Pythagorean Theorem, which students remember from Geometry.
Bloom's Levels: Analyze
Webb's DOK: 3

## Topic 2: Linear Functions

## Engaging Experience 1

Title: Writing Equations of Lines Line-Up
Suggested Length of Time: 15 minutes
Standards Addressed
Priority:

- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.
Supporting:
- NMP.FF. 2 Use multiple representations of functions to interpret and describe how two quantities change together.
Detailed Description/Instructions: Students will be given a card that has given information about their line. This might be a point and a slope, two points, or information about a parallel/perpendicular line. Students will find the equation of their line, written in slope-intercept form. Afterwards, they will line up from smallest $y$-intercept to largest $y$-intercept. Count the number of mistakes (if any) and have students complete the problem of the person standing next to them to check for accuracy.
Bloom's Levels: Analyze
Webb's DOK: 3


## Topic 3: Functions

## Engaging Experience 1

Title: Marbleslides Lines in Desmos
Suggested Length of Time: 20 minutes

## Standards Addressed

Priority:

- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.
Supporting:
- NMP.FF. 3 Measure, compute, describe, and interpret rates of change of quantities embedded in multiple representations.
- ISTE-COMPUTATIONAL THINKER.5.B - collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.
Detailed Description/Instructions: Students will follow prompts in a Desmos activity that allows them to construct lines that intersect various "stars." Students will need to create multiple functions to make the scenario work and also keep mindful about domain restrictions, although they do not need to be fluent with the concept of domain at this point in time.
Bloom's Levels: Analyze
Webb's DOK: 3


## Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will complete a linear regression project. Students will analyze the linear relationship (correlation) between two variables of their choosing. The project consists of researching data, analysis, and presenting the data and analysis in a formal report.

## Summary of Engaging Learning Experiences for Topics

| Topic | Engaging Experience Title | Description | Suggested <br> Length of Time |
| :---: | :---: | :---: | :---: |
| Coordinate Geometry | Deriving formulas for distance and midpoint using the Pythagorean Theorem | Instead of giving students the formulas they will need for distance and midpoint, we will prove them using the Pythagorean Theorem, which students remember from Geometry. | 15 minutes |
| Linear <br> Functions | Writing <br> Equations of Lines Line-Up | Students will be given a card that has given information about their line. This might be a point and a slope, two points, or information about a parallel/perpendicular line. Students will find the equation of their line, written in slope-intercept form. Afterwards, they will line up from smallest y-intercept to largest y -intercept. Count the number of mistakes (if any) and have students complete the problem of the person standing next to them to check for accuracy. | 15 minutes |
| Functions | Marbleslides Lines in Desmos | Students will follow prompts in a Desmos activity that allows them to construct lines that intersect various "stars." Students will need to create multiple functions to make the scenario work and also keep mindful about domain restrictions, although they do not need to be fluent with the concept of domain at this point in time. | 20 minutes |

## Unit 3: Systems

Subject: College Algebra
Grade: 10, 11, 12
Name of Unit: Systems
Length of Unit: 3 class periods
Overview of Unit: In this unit students will solve systems of linear equations in two and three variables.

Priority Standards for unit:

- Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.
- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.


## Supporting Standards for unit:

- ISTE-KNOWLEDGE COLLECTOR.3.D - build knowledge by actively exploring realworld issues and problems, developing ideas and theories and pursuing answers and solutions.
- ISTE-COMPUTATIONAL THINKER.5.A - formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.

| Unwrapped Concepts <br> (Students need to know) | Unwrapped Skills <br> (Students need to be able to do) | Bloom's <br> Taxonomy <br> Levels | Webb's <br> DOK |
| :---: | :---: | :---: | :---: |
| systems of equations that may <br> include non-linear equations and <br> inequalities. | Create | Create | 3 |
| systems of equations that may <br> include non-linear equations and <br> inequalities. | Solve | Apply | 2 |
| key characteristics of functions <br> represented graphically, with <br> tables and with algebraic <br> symbolism to solve problems. | Identify | Understand | 2 |
| key characteristics of functions <br> represented graphically, with <br> tables and with algebraic | Interpret | Analyze | 3 |

## Essential Questions:

1. How do we solve a linear system of two and three variables?
2. What are the solution types of a linear system, and how are they used to classify the system?

## Enduring Understanding/Big Ideas:

1. Use various strategies including graphing, substitution, and linear combination
2. The solutions can be independent and consistent, dependent and consistent, or inconsistent, and they are used to describe any linear system.

Unit Vocabulary:

| Academic Cross-Curricular Words | Content/Domain Specific |
| :---: | :---: |
|  | Substitution |
|  | Linear Combination |
|  | Consistent |
|  | Inconsistent |
|  | Independent |
|  | Dependent |
|  | Commutativity |

Resources for Vocabulary Development: textbook

## Topic 1: Solve and Apply

## Engaging Experience 1

Title: The Tortoise and Hare Activity
Suggested Length of Time: 45 minutes
Standards Addressed
Priority:

- Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.
- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.


## Supporting:

- ISTE-COMPUTATIONAL THINKER.5.A - formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.
Detailed Description/Instructions: The Tortoise and the Hare finally have their long-awaited rematch. The Tortoise gets a 1,000-foot lead and runs at 9 inches per second. The Hare begins at the starting line and runs at a rate of 6 feet per second. There is also a rat in this race. The Rat starts 1,200 feet ahead of the Hare and runs back towards the starting line at a rate of 2 feet per second.

In this problem, students extract data from a story in order to write, manipulate, and graph systems of equations. It offers students a context to understand the relationships among data, equations, graphs and solutions.
Bloom's Levels: Create and Apply
Webb's DOK: 3, 2

## Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will research two cars, given specific guidelines. One is an older model, cheap, sports car that gets poor gas mileage, the other a newer more economical, more expensive vehicle. Students will research the cost and gas mileage. They will then model this information with linear equations, graph and determine at what point in time the cost of the two vehicles would intersect. For the final product they would then have to summarize by determining which car would be the best and give mathematically supported reasons.

## Summary of Engaging Learning Experiences for Topics

$\left.$| Topic | Engaging <br> Experience <br> Title | Description | Suggested <br> Length of <br> Time |
| :---: | :---: | :---: | :---: |
| Solve and | The Tortoise <br> apply Hare <br> Activity | The Tortoise and the Hare finally have their <br> long-awaited rematch. The Tortoise gets a <br> 1000 -foot lead and runs at 9 inches per second. <br> The Hare begins at the starting line and runs at | 45 minutes |
| a rate of 6 feet per second. There is also a rat in |  |  |  |
| this race. The Rat starts 1,200 feet ahead of the |  |  |  |
| Hare and runs back towards the starting line at |  |  |  |
| a rate of 2 feet per second. In this problem, |  |  |  |
| students extract data from a story in order to |  |  |  |
| write, manipulate, and graph systems of |  |  |  |
| equations. It offers students a context to |  |  |  |
| understand the relationships among data, |  |  |  |
| equations, graphs and solutions. |  |  |  |$\quad \right\rvert\,$

## Unit 4: Exponents and Polynomials

Subject: College Algebra
Grade: 10, 11, 12
Name of Unit: Exponents and Polynomials
Length of Unit: 5 class periods
Overview of Unit: In this unit students will simplify expressions using the rules of exponents.
Students will also factor and solve quadratic (and quadratic type) equations.

## Priority Standards for unit:

- Alg1.SSE.A.2: Analyze the structure of polynomials to create equivalent expressions or equations.


## Supporting Standards for unit:

- Alg2.FM.A.1: Create functions and use them to solve applications of quadratic and exponential function model problems.
- ISTE-COMPUTATIONAL THINKER.5.C - break problems into component parts, extract key information, and develop descriptive models to understand complex systems or facilitate problem-solving.
- ISTE-GLOBAL COLLABORATOR.7.C - contribute constructively to project teams, assuming various roles and responsibilities to work effectively toward a common goal.
- ISTE-INNOVATIVE DESIGNER.4.B - select and use digital tools to plan and manage a design process that considers design constraints and calculated risks.

| Unwrapped Concepts <br> (Students need to know) | Unwrapped Skills <br> (Students need to be able to do) | Bloom's <br> Taxonomy <br> Levels | Webb's <br> DOK |
| :---: | :---: | :---: | :---: |
| the structure of polynomials to <br> create equivalent expressions or <br> equations. | Analyze |  |  |

## Essential Questions:

1. How do you simplify expressions using exponents?
2. How do you write a polynomial in completely factored form?
3. What is the zero-factor property and how do you use it to solve polynomial equations?

## Enduring Understanding/Big Ideas:

1. Use the properties of exponents.
2. Use various strategies of factoring including: GCF, difference of squares, perfect square trinomials, trinomials, and sum/difference of cubes.
3. When the product of two real numbers is zero, at least one of them is zero. This can be used to solve for each factor.

Unit Vocabulary:

| Academic Cross-Curricular Words | Content/Domain Specific |
| :---: | :---: |
| Properties/rules/laws | Simplest form |
| Coefficient | Factor |
|  | Polynomial |
| Base |  |
|  | Degree |
| Monomial |  |
| Binomial |  |
|  | Trinomial |
|  | Perfect Square Trinomial |
| Greatest Common Factor |  |
| FOIL |  |
|  | Prime |
|  | Zero-Product Property |

Resources for Vocabulary Development: textbook

## Topic 1: Simplify Polynomials

## Engaging Experience 1

Title: Polynomial Operation Mix-And-Match Activity
Suggested Length of Time: 20 minutes
Standards Addressed
Priority:

- Alg1.SSE.A.2: Analyze the structure of polynomials to create equivalent expressions or equations.


## Supporting:

- Alg2.FM.A.1: Create functions and use them to solve applications of quadratic and exponential function model problems.
Detailed Description/Instructions: Students will be given a note card with a polynomial expression on it, and they will be expected to analyze the degree, number of terms, and leading coefficient. In the first round, students will organize themselves into groups by degree, answering questions involving adding and subtracting polynomials. In the second round, students will organize themselves by number of terms and will multiply polynomials. In the third round, students will find groups based on the leading coefficient and will simplify expressions in quadratic form and using division.
Bloom's Levels: Analyze
Webb's DOK: 2


## Topic 2: Factoring

## Engaging Experience 1

Title: Factoring Using Algebra Tiles
Suggested Length of Time: 10 minutes
Standards Addressed
Priority:

- Alg1.SSE.A.2: Analyze the structure of polynomials to create equivalent expressions or equations.


## Supporting:

- Alg2.FM.A.1: Create functions and use them to solve applications of quadratic and exponential function model problems.
- ISTE-COMPUTATIONAL THINKER.5.C - break problems into component parts, extract key information, and develop descriptive models to understand complex systems or facilitate problem-solving.
Detailed Description/Instructions: Students will be given the task of creating a rectangle given algebra tiles or creating their own tiles using Smart Note book. For example, the equation $\mathrm{x} 2+8 \mathrm{x}+15$ would be represented by one blue square, 8 green "rows of $x$ " and 15 green single tiles. Using those tiles, students will attempt to create a rectangle. In this activity, students will see the relationship between factoring and FOILing and start to learn what it means to both be a perfect square trinomial and complete the square.
Bloom's Levels: Analyze
Webb's DOK: 2


## Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will pair up to design a swimming pool complex using specific guidelines and a budget. Students will be required to multiply polynomials, factor, and solve quadratics by factoring in a real-world scenario.

Summary of Engaging Learning Experiences for Topics

| Topic | Engaging <br> Experience <br> Title | Description | Suggested <br> Length of <br> Time |
| :---: | :---: | :---: | :---: |
| Simplify <br> Polynomials | Polynomial <br> Operation Mix- <br> And-Match <br> Activity | Students will be given a note card with a <br> polynomial expression on it, and they will be <br> expected to analyze the degree, number of <br> terms, and leading coefficient. In the first <br> round, students will organize themselves into <br> groups by degree, answering questions <br> involving adding and subtracting polynomials. <br> In the second round, students will organize <br> themselves by number of terms and will <br> multiply polynomials. In the third round, <br> students will find groups based on the leading <br> coefficient and will simplify expressions in <br> quadratic form and using division. | 20 minutes |
| Factoring | Factoring Using <br> Algebra Tiles | Students will be given the task of creating a <br> rectangle given algebra tiles or creating their <br> own tiles using Smart Note book. For <br> example, the equation x2+8x+15 would be <br> represented by one blue square, 8 green "rows <br> of x" and 15 green single tiles. Using those <br> tiles, students will attempt to create a <br> rectangle. In this activity, students will see the <br> relationship between factoring and FOILing <br> and start to learn what it means to both be a <br> perfect square trinomial and complete the <br> square. | 10 minutes |

## Unit 5: Rationals

Subject: College Algebra
Grade: 10, 11, 12
Name of Unit: Rationals
Length of Unit: 6 class periods
Overview of Unit: In this unit students will simplify, add, subtract, multiply, and divide rational functions. Students will also solve rational equations.

## Priority Standards for unit:

- Alg2.REI.A.2: Solve rational equations where numerators and denominators are polynomials and where extraneous solutions may result.
- Alg2.APR.A.4: Add, subtract, multiply and divide rational expressions.


## Supporting Standards for unit:

- Alg2.APR.A.2: Understand the Remainder Theorem and use it to solve problems.
- Alg2.APR.A.3: Find the least common multiple of two or more polynomials.
- ISTE-EMPOWERED LEARNER1.C - use technology to seek feedback that informs and improves their practice and to demonstrate their learning in a variety of ways.
- ISTE-GLOBAL COLLABORATOR.7.C - contribute constructively to project teams, assuming various roles and responsibilities to work effectively toward a common goal.

| Unwrapped Concepts <br> (Students need to know) | Unwrapped Skills <br> (Students need to be able to do) | Bloom's <br> Taxonomy <br> Levels | Webb's <br> DOK |
| :---: | :---: | :---: | :---: |
| rational equations where <br> numerators and denominators are <br> polynomials and where extraneous <br> solutions may result |  |  |  |
| rational expressions | Add, subtract, multiply and divide | Apply | 1 |

## Essential Questions:

1. How do you simplify rational expressions?
2. Why is the domain affected by rational functions?
3. Why are the domain restrictions in rational equations important?
4. How do you solve rational equations and use them to model real-world applications?

## Enduring Understanding/Big Ideas:

1. Add, subtract, multiply, and divide using the cancellation property and common denominators.
2. A domain restriction occurs when there are values for $x$ that make the function undefined. Rational functions have undefined values because they make the denominator equal to zero.
3. Domain restrictions create potentially extraneous solutions.
4. Find the domain restrictions, multiply by the LCD, and solve for the variable. Check for extraneous solutions by analyzing the domain restrictions.

Unit Vocabulary:

| Academic Cross-Curricular Words | Content/Domain Specific |
| :---: | :---: |
| Isolate variable | Domain restrictions |
|  | Complex fractions |
|  | Least Common Denominator |
| Least Common Divisor |  |
| Extraneous |  |
| Cancellation Property |  |

Resources for Vocabulary Development: textbook

## Topic 1: Simplify Rational Expressions

## Engaging Experience 1

Title: Quadrant Partners Activity
Suggested Length of Time: 20 minutes

## Standards Addressed

Priority:

- Alg2.APR.A.4: Add, subtract, multiply and divide rational expressions.


## Supporting:

- Alg2.APR.A.3: Find the least common multiple of two or more polynomials.
- ISTE-EMPOWERED LEARNER1.C - use technology to seek feedback that informs and improves their practice and to demonstrate their learning in a variety of ways.
Detailed Description/Instructions: Students will find four partners around the room (a different individual for each of their quadrants). In their partnership, students will work together to solve a problem put the solution in an online document such as the collaboration space on One Note, and the teacher will ensure each group has the correct solution, addressing any misconceptions as they arise. This process will be repeated for each of the four quadrant partners.
Bloom's Levels: Apply
Webb's DOK: 1


## Engaging Experience 2

Title: RallyCoach Activity
Suggested Length of Time: 10 minutes
Standards Addressed
Priority:

- Alg2.APR.A.4: Add, subtract, multiply and divide rational expressions.


## Supporting:

- Alg2.APR.A.3: Find the least common multiple of two or more polynomials.

Detailed Description/Instructions: Students will find a partner and complete four problems. Two of the problems will be completed by Partner A, with Partner B checking for accuracy. Similarly, two problems will be completed by Partner B, with Partner A checking for accuracy. This accountability piece will hold each partner accountable, even if he or she is not currently working a problem.
Bloom's Levels: Apply
Webb's DOK: 1

## Topic 2: Solve and Apply

## Engaging Experience 1

Title: Round Table Activity
Suggested Length of Time: 45 minutes
Standards Addressed
Priority:

- Alg2.REI.A.2: Solve rational equations where numerators and denominators are polynomials and where extraneous solutions may result.
Supporting:
- Alg2.APR.A.2: Understand the Remainder Theorem and use it to solve Problems.
- ISTE-GLOBAL COLLABORATOR.7.C - contribute constructively to project teams, assuming various roles and responsibilities to work effectively toward a common goal.
Detailed Description/Instructions: Students will be placed in groups of four by the teacher and presented with a problem involving modeling a real-world scenario using rational equations. Each student will have one of four unique roles. He or she will either define the variables/ provide a diagram, set up the equation, solve the equation, or effectively communicate the solution to the problem. This process will be repeated four times so that each team member can experience each role in the group, with the teacher checking the solution before providing the next problem to each group.
Bloom's Levels: Analyze
Webb's DOK: 3


## Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will be given the choice of a task (work out simple math problems, shoot baskets, stack blocks, etc.). Students will then individually complete their task and record the time it took for them to complete given task. Students will then pair up and, based on their individual times, calculate how long it would take them to complete the task together. Students will then complete the task together, record the time it took for them to complete the given task together and compare their results.

## Summary of Engaging Learning Experiences for Topics

| Topic | Engaging Experience Title | Description | Suggested <br> Length of Time |
| :---: | :---: | :---: | :---: |
| Simplify <br> Rational Expressions | Quadrant <br> Partners <br> Activity | Students will find four partners around the room (a different individual for each of their quadrants). In their partnership, students will work together to solve a problem put the solution in an online document such as the collaboration space on One Note, and the teacher will ensure each group has the correct solution, addressing any misconceptions as they arise. This process will be repeated for each of the four quadrant partners. | 20 minutes |
| Simplify <br> Rational Expressions | RallyCoach Activity | Students will find a partner and complete four problems. Two of the problems will be completed by Partner A, with Partner B checking for accuracy. Similarly, two problems will be completed by Partner B, with Partner A checking for accuracy. This accountability piece will hold each partner accountable, even if he or she is not currently working a problem. | 10 minutes |
| Solve and Apply | Round Table Activity | Students will be placed in groups of four by the teacher and presented with a problem involving modeling a real-world scenario using rational equations. Each student will have one of four unique roles. He or she will either define the variables/ provide a diagram, set up the equation, solve the equation, or effectively communicate the solution to the problem. This process will be repeated four times so that each team member can experience each role in the group, with the teacher checking the solution before providing the next problem to each group. | 45 minutes |

## Unit 6: Radicals

Subject: College Algebra
Grade: 10, 11, 12
Name of Unit: Radicals
Length of Unit: 6 class periods
Overview of Unit: In this unit students will work with radicals and rational exponents. They will also be introduced to complex numbers and operations with complex numbers.

## Priority Standards for unit:

- Alg2.NQ.A.1: Extend the system of powers and roots to include rational exponents.
- Alg2.NQ.A.2: Create and recognize equivalent expressions involving radical and exponential forms of expressions.
- Alg2.NQ.A.3: Add, subtract, multiply and divide radical expressions.
- Alg2.NQ.A.4: Solve equations involving rational exponents and/or radicals and identify situations where extraneous solutions may result.


## Supporting Standards for unit:

- Alg2.NQ.B.1: Represent complex numbers.
- Alg2.NQ.B.2: Add, subtract, multiply and divide complex numbers.
- Alg2.IF.A.2: Translate between equivalent forms of functions.
- NMP.FF. 2 Use multiple representations of functions to interpret and describe how two quantities change together.
- ISTE-KNOWLEDGE COLLECTOR.3.B - evaluate the accuracy, perspective, credibility and relevance of information, media, data or other resources.
- ISTE-EMPOWERED LEARNER1.D - understand the fundamental concepts of technology operations, demonstrate the ability to choose, use and troubleshoot current technologies and are able to transfer their knowledge to explore emerging technologies.

| Unwrapped Concepts <br> (Students need to know) | Unwrapped Skills <br> (Students need to be able to do) | Bloom's <br> Taxonomy <br> Levels | Webb's <br> DOK |
| :---: | :---: | :---: | :---: |
| the system of powers and roots to <br> include rational exponents. | Extend | Understand | 4 |
| equivalent expressions involving <br> radical and exponential forms of <br> expressions. | Create | Create | 1 |
| equivalent expressions involving <br> radical and exponential forms of <br> expressions. | Recognize | Create | 2 |


| radical expressions. | Add, subtract, multiply, and <br> divide | Apply | 1 |
| :---: | :---: | :---: | :---: |
| equations involving rational <br> exponents and/or radicals | Solve | Apply | 2 |
| situations where extraneous <br> solutions may result | Identify | Analyze | 3 |

## Essential Questions:

1. How do you simplify radicals?
2. How do you combine radicals using the basic operations?
3. How do you rewrite radicals using rational exponents?
4. How do you solve equations involving radicals?
5. What is a complex number and how do you simplify complex numbers?

## Enduring Understanding/Big Ideas:

1. You can simplify radicals by writing a radical in simplest form including the proper use of absolute values.
2. Adding, subtracting, multiplying and dividing radicals, including rationalizing the denominator.
3. Rewriting a radical using rational exponents, and rewriting a number with a rational exponent as a radical.
4. Isolate the variable by applying the order of operations backwards and checking for extraneous solutions.
5. Define a complex number and do the basic mathematical operations with complex numbers.

## Unit Vocabulary:

| Academic Cross-Curricular Words | Content/Domain Specific |
| :---: | :---: |
|  | Radical |
|  | Rational Exponents |
| Rationalize |  |
|  | Extraneous |
|  | Complex Numbers |
| Conjugate |  |
|  | Radicand |
| Index |  |
|  | Root |
|  | Square Root |

Resources for Vocabulary Development: Textbook

## Topic 1: Simplifying Radicals

## Engaging Experience 1

Title: Guided Practice
Suggested Length of Time: 15 minutes
Standards Addressed
Priority:

- ALG2.NQ.A.1: Extend the system of powers and roots to include rational exponents.
- ALG2.NQ.A.2: Create and recognize equivalent expressions involving radical and exponential forms of expressions.
Supporting:
- Alg2.IF.A.2: Translate between equivalent forms of functions.

Detailed Description/Instructions: Students will review the process of simplifying radicals by performing various examples of simplifying radicals on a personal whiteboard (or using their laptop as a whiteboard).
Bloom's Levels: Understand, Create, Create
Webb's DOK: 4, 1, 2

## Topic 2: Solving Radical Equations

## Engaging Experience 1

Title: Who is Right?
Suggested Length of Time: 10 minutes

## Standards Addressed

Priority:

- Alg2.NQ.A.4: Solve equations involving rational exponents and/or radicals and identify situations where extraneous solutions may result.


## Supporting:

- NMP.FF. 2 Use multiple representations of functions to interpret and describe how two quantities change together.
- ISTE-KNOWLEDGE COLLECTOR.3.B - evaluate the accuracy, perspective, credibility and relevance of information, media, data or other resources.
Detailed Description/Instructions: Give students a problem (sqrt of $x=-3$ ). Show two students solutions - Carlos says the answer is 9 because you are supposed to square both sides, Andrea says there is no solution, have students discuss who is right and why - be sure to emphasize the importance of checking for extraneous solutions. Then move on to a more difficult example ( $6=$ $x+\operatorname{sqrt}(x))$. Have students solve them and compare their solutions with a partner.
Bloom's Levels: Apply
Webb's DOK: 1


## Topic 3: Complex Numbers

## Engaging Experience 1

Title: Who is Right?
Suggested Length of Time: 10-15 minutes
Standards Addressed
Priority:

- Alg2.NQ.A.3: Add, subtract, multiply and divide radical expressions


## Supporting:

- Alg2.NQ.B.1: Represent complex numbers.
- Alg2.NQ.B.2: Add, subtract, multiply and divide complex numbers.
- ISTE-KNOWLEDGE COLLECTOR.3.B - evaluate the accuracy, perspective, credibility and relevance of information, media, data or other resources.
Detailed Description/Instructions: After reviewing the definition of a complex number present students with the following problem: $\operatorname{sqrt}(-4) \times \operatorname{sqrt}(-4)=4$. Discuss as a class or in small groups whether this is correct or incorrect and why. Then go on to explain, if necessary, that if sqrt(3) $x$ $\operatorname{sqrt}(3)=3$, shouldn't the sqrt( -4 ) x sqrt( -4 ) $=-4$ ? Using complex numbers show why this works.
Bloom's Levels: Apply
Webb's DOK: 1


## Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will access the graphing calculator features on Desmos. They will input various nth roots of the nth powers to notice patterns. Through discussion and collaboration, we will determine when the graphs are strictly located above the $x$-axis. Then, we will reverse the process to see when extraneous solutions are needed.

Summary of Engaging Learning Experiences for Topics

| Topic | Engaging <br> Experience Title | Description | Suggested <br> Length of Time |
| :---: | :---: | :---: | :---: |
| Simplifying Radicals | Guided <br> Practice | Students will review the process of simplifying radicals by performing various examples of simplifying radicals on a personal whiteboard (or using their laptop as a whiteboard). | 15 minutes |
| Solving Radical <br> Equations | Who is Right? | Give students a problem (sqrt of $x=-3$ ). Show two students solutions - Carlos says the answer is 9 because you are supposed to square both sides, Andrea says there is no solution, have students discuss who is right and why - be sure to emphasize the importance of checking for extraneous solutions. Then move on to a more difficult example ( $6=x+\operatorname{sqrt}(x)$ ). Have students solve them and compare their solutions with a partner. | 10 minutes |
| Complex <br> Numbers | Who is Right? | After reviewing the definition of a complex number present students with the following problem: sqrt( -4 ) x sqrt( -4 ) $=4$. Discuss as a class or in small groups whether this is correct or incorrect and why. Then go on to explain, if necessary, that if $\operatorname{sqrt}(3) \times \operatorname{sqrt}(3)=3$, shouldn't the sqrt(-4) $x$ sqrt( -4 ) $=-4$ ? Using complex numbers show why this works. | $\begin{gathered} 10-15 \\ \text { minutes } \end{gathered}$ |

## Unit 7: Quadratics

Subject: College Algebra
Grade: 10, 11, 12
Name of Unit: Quadratics
Length of Unit: 6 class periods
Overview of Unit: In this unit students will solve quadratic (and quadratic type) functions, analyze the graph of a quadratic function and solve quadratic inequalities.

## Priority Standards for unit:

- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.


## Supporting Standards for unit:

- Alg2.IF.A.2: Translate between equivalent forms of functions.
- Alg2.FM.A.1: Create functions and use them to solve applications of quadratic and exponential function model problems.
- NMP.FF. 2 Use multiple representations of functions to interpret and describe how two quantities change together.
- ISTE-KNOWLEDGE COLLECTOR.3.D - build knowledge by actively exploring realworld issues and problems, developing ideas and theories and pursuing answers and solutions.
- ISTE-COMPUTATIONAL THINKER.5.B - collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problemsolving and decision-making.
- Alg1.SSE.A.3: Choose and produce equivalent forms of a quadratic expression or equations to reveal and explain properties.
- Find the zeros of a quadratic function by rewriting it in factored form.
- Find the maximum or minimum value of a quadratic function by completing the square.

| Unwrapped Concepts <br> (Students need to know) | Unwrapped Skills <br> (Students need to be <br> able to do) | Bloom's <br> Taxonomy <br> Levels | Webb's <br> DOK |
| :---: | :---: | :---: | :---: |
| key characteristics of functions represented <br> graphically, with tables and with algebraic <br> symbolism to solve problems. | Identify | Understand | 2 |
| key characteristics of functions represented <br> graphically, with tables and with algebraic <br> symbolism to solve problems. | Interpret | Analyze | 3 |

## Essential Questions:

1. How do you solve a quadratic function?
2. How do you use the graph of a quadratic function to find the maximum/minimum values?
3. How do you solve a quadratic inequality?
4. How do you write a quadratic equation from given roots?
5. How do you solve other polynomials in quadratic form?

## Enduring Understanding/Big Ideas:

1. Solve a quadratic function using a variety of techniques including factoring, the quadratic formula and completing the square.
2. Use the vertex to graph a quadratic function and find the maximum or minimum value of that function.
3. Solve a quadratic inequality by graphing and using a table and then communicating the solution using interval notation.
4. Work backwards to write a quadratic equation from given roots.
5. Solving quadratic type equations using substitution.

## Unit Vocabulary:

| Academic Cross-Curricular Words | Content/Domain Specific |
| :---: | :---: |
| Roots | Quadratic |
| Optimization (Maximum/Minimum) | Discriminant |
| Intercepts | Quadratic inequalities |
|  | Factoring |
| Parabola |  |
| Vertex |  |

Resources for Vocabulary Development: Textbook

## Topic 1: Polynomial Functions

## Engaging Experience 1

Title: Generate the quadratic formula by completing the square
Suggested Length of Time: 10 minutes

## Standards Addressed

Priority:

- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.


## Supporting:

- Alg2.IF.A.2: Translate between equivalent forms of functions
- Alg2.FM.A.1: Create functions and use them to solve applications of quadratic and exponential function model problems.
- Alg1.SSE.A.3: Choose and produce equivalent forms of a quadratic expression or equations to reveal and explain properties.
Find the zeros of a quadratic function by rewriting it in factored form.
Find the maximum or minimum value of a quadratic function by completing the square.
- ISTE-KNOWLEDGE COLLECTOR.3.D - build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.
Detailed Description/Instructions: Students can complete this on a white board: Have the students write out a quadratic equation in standard form (with values for $\mathrm{a}, \mathrm{b}$, and c where a does not equal 1). Have them solve this equation by completing the square. Now have them write out $a x^{\wedge} 2+b x+c=0$ and solve this by completing the square. If they do it correctly, they should generate the quadratic formula.
Bloom's Levels: Understand, Analyze
Webb's DOK: 2, 3


## Topic 2: Solving and Applying

## Engaging Experience 1

Title: Making a connection
Suggested Length of Time: 20 minutes Standards Addressed

Priority:

- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.


## Supporting:

- NMP.FF. 2 Use multiple representations of functions to interpret and describe how two quantities change together.
- ISTE-COMPUTATIONAL THINKER.5.B - collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.
Detailed Description/Instructions: Give the students a quadratic inequality and have them graph it on their graphing calculator (or use Desmos), demonstrate the solution using interval notation. Give them a few more examples with a variety of solution types and a variety of types of quadratic functions (some in factored form). Discuss what conclusions can be made about the solutions (they should notice that the critical points come from the roots of the quadratic equations). Now demonstrate the test-point method as another way of solving a quadratic inequality without graphing on a coordinate plane.
Bloom's Levels: Understand, Analyze
Webb's DOK: 2, 3


## Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) "Match My Curve!" activity on Desmos. In this activity, students develop their understanding of standard, factored, and vertex forms of quadratic functions by matching parabolas (as closely as they can) to curves in images from the real world.

Summary of Engaging Learning Experiences for Topics

| Topic | $\begin{array}{c}\text { Engaging } \\ \text { Experience } \\ \text { Title }\end{array}$ | Description | $\begin{array}{c}\text { Suggested } \\ \text { Length of } \\ \text { Time }\end{array}$ |
| :---: | :---: | :---: | :---: |
| $\begin{array}{c}\text { Polynomial } \\ \text { Functions }\end{array}$ | $\begin{array}{c}\text { Generate the } \\ \text { quadratic } \\ \text { formula by } \\ \text { completing } \\ \text { the square }\end{array}$ | $\begin{array}{c}\text { Students can complete this on a white board: } \\ \text { Have the students write out a quadratic } \\ \text { equation in standard form (with values for a, b, } \\ \text { and c where a does not equal 1). Have them } \\ \text { solve this equation by completing the square. } \\ \text { Now have them write out ax^2 + bx + c = 0 } \\ \text { and solve this by completing the square. If }\end{array}$ | 10 minutes |
| they do it correctly, they should generate the |  |  |  |
| quadratic formula. |  |  |  |$]$

## Unit 8: Functions

Subject: College Algebra
Grade: 10, 11, 12
Name of Unit: Functions
Length of Unit: 9 class periods
Overview of Unit: Students will graph a variety of functions use domain, range, tables, and transformations. Students will use function notation to combine functions, find average rate of change, and determine if a function has symmetry. Students will find inverses to a function algebraically, graphically and numerically.

## Priority Standards for unit:

- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.
- Alg2.BF.A.1: Create new functions by applying the four arithmetic operations and composition of functions (modifying the domain and range as necessary).
- Alg2.BF.A.2: Derive inverses of functions, and compose the inverse with the original function to show that the functions are inverses.


## Supporting Standards for unit:

- Alg2.BF.A.3: Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for linear, quadratic, cubic, square and cube root, absolute value, exponential and logarithmic functions.
- NMP.FF. 2 Use multiple representations of functions to interpret and describe how two quantities change together.
- ISTE-EMPOWERED LEARNER1.D - understand the fundamental concepts of technology operations, demonstrate the ability to choose, use and troubleshoot current technologies and are able to transfer their knowledge to explore emerging technologies
- ISTE-KNOWLEDGE COLLECTOR.3.D - build knowledge by actively exploring realworld issues and problems, developing ideas and theories and pursuing answers and solutions.

| Unwrapped Concepts <br> (Students need to know) | Unwrapped Skills <br> (Students need to be <br> able to do) | Bloom's <br> Taxonomy <br> Levels | Webb's <br> DOK |
| :---: | :---: | :---: | :---: |
| key characteristics of functions represented <br> graphically, with tables and with algebraic <br> symbolism to solve problems. | Identify | Understand | 2 |


| key characteristics of functions represented <br> graphically, with tables and with algebraic <br> symbolism to solve problems. | Interpret | Analyze | 3 |
| :---: | :---: | :---: | :---: |
| Create new functions by applying the four <br> arithmetic operations and composition of <br> functions (modifying the domain and range <br> as necessary). | Create | Create | 2 |
| Create new functions by applying the four <br> arithmetic operations and composition of <br> functions (modifying the domain and range <br> as necessary). |  |  |  |
| Derive inverses of functions, and compose <br> the inverse with the original function to show <br> that the functions are inverses. | Apply | Apply | 1 |
| Derive inverses of functions, and compose <br> the inverse with the original function to show <br> that the functions are inverses. | Derive | Apply | 2 |

## Essential Questions:

1. How can you represent a function in a variety of ways?
2. How can you find the domain and range of a function and use that to graph a given function?
3. How can you analyze the graph of a function?
4. How do you combine functions?
5. How do you find the inverse of a function?

## Enduring Understanding/Big Ideas:

1. Represent a function verbally, algebraically, graphically and numerically, using function notation when appropriate.
2. Determine the domain and range of a function, use the information to graph a variety of functions including piecewise functions.
3. Using the graph of a function to apply transformations, find optimal values and determine the average rate of change.
4. Use the basic operations to combine functions to obtain new functions and finding the composition of functions.
5. Finding the inverse of $1-1$ functions and proving that functions are inverses of each other.

Unit Vocabulary:

| Academic Cross-Curricular Words | Content/Domain Specific |
| :---: | :---: |
| Function | Domain |
| Range |  |
| Independent Variable | Piecewise Function |
| Dependent Variable | Even and Odd Functions |
| Net Change | Composition |
| Maxima | One to One |
| Minima | Horizontal Line Test |
| Average Rate of Change |  |
| Transformations |  |
| Inverse |  |

Resources for Vocabulary Development: Textbook

## Topic 1: Functions and Graphs

## Engaging Experience 1

Title: Just Move it - Desmos
Suggested Length of Time: 45 minutes
Standards Addressed
Priority:

- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.


## Supporting:

- Alg2.BF.A.3: Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for linear, quadratic, cubic, square and cube root, absolute value, exponential and logarithmic functions.
- NMP.FF. 2 Use multiple representations of functions to interpret and describe how two quantities change together.
- ISTE-EMPOWERED LEARNER1.D - understand the fundamental concepts of technology operations, demonstrate the ability to choose, use and troubleshoot current technologies and are able to transfer their knowledge to explore emerging technologies.
Detailed Description/Instructions: Students will complete the Move It - Transformations activity on Desmos.
Bloom's Levels: Understand and Analyze
Webb's DOK: 2, 3


## Topic 2: Analyzing Functions

## Engaging Experience 1

Title: Cell Phone Activity
Suggested Length of Time: 30 minutes

## Standards Addressed

Priority:

- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.


## Supporting:

- NMP.FF. 2 Use multiple representations of functions to interpret and describe how two quantities change together.
- ISTE-KNOWLEDGE COLLECTOR.3.D - build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.
Detailed Description/Instructions: After being introduced to the idea of a piecewise function students will be given a cell phone scenario with 2-3 different plans (for example plan A: \$30 monthly fee, 450 anytime minutes, $\$ .45$ for each additional minute, plan B: $\$ 45$ monthly fee, 900 anytime minutes, $\$ .30$ for each additional minute). Students will be given a task to create a piecewise function to represent each scenario, then answer questions about which is the best plan if you use 400 minutes each month, 475,600 , etc., verifying this both graphically and algebraically.
Bloom's Levels: Understand, Analyze
Webb's DOK: 2, 3


## Topic 3: Combining Functions

## Engaging Experience 1

Title: Chain Reaction
Suggested Length of Time: 15 minutes

## Standards Addressed

Priority:

- Alg2.BF.A.1: Create new functions by applying the four arithmetic operations and composition of functions (modifying the domain and range as necessary).
Supporting:
- NMP.FF. 2 Use multiple representations of functions to interpret and describe how two quantities change together.
Detailed Description/Instructions: Students will be divided into groups of 4 or 5 and labeled with a number 1-5 (for groups of 4 one person will go twice). The teacher will prepare 5 functions (using a variety of ones discussed - quadratic, linear, square root, etc.) and print these same 5 functions on note cards for each of the groups in the class. Each group will receive the set of 5 functions and distribute them to their group members in random order. The teacher will then call out a number, for example -1 . The 1 student in each group will evaluate his/her function at that value, the 2 students will then evaluate his/her function at the answer to student 1 's function. This will continue on until all 5 functions have been evaluated. Answers will be checked, and students will then have an opportunity to go back and fix their chain if they were wrong. This is a good lead into discussing the composition of functions.
Bloom's Levels: Apply
Webb's DOK: 2


## Topic 4: Inverse Functions

## Engaging Experience 1

Title: Inverse function partner share
Suggested Length of Time: 30 minutes

## Standards Addressed

Priority:

- Alg2.BF.A.2: Derive inverses of functions, and compose the inverse with the original function to show that the functions are inverses.


## Supporting:

- NMP.FF. 2 Use multiple representations of functions to interpret and describe how two quantities change together
Detailed Description/Instructions: The teacher will begin by writing a function on the board (for example $3 x^{\wedge} 2+1$ ) and as a class they should generate the sequence of steps needed to evaluate the function ( 1 - square the input, 2 - multiply by 3 , 3 -add 1 ). From here the class will divide into teams of 2 . Each team of two will be given two different functions that are not inverses of each other, however another team should have functions that are inverses of another team's functions. Students should write out the sequence of steps for each of their functions and provide 3 ordered pairs which satisfy their functions. Once each team is done, they should seek out a team that mirrors their steps. Once they have found their other half, they should discuss what they notice about their functions (the steps are in the opposite order - steps are "undone", they should notice their ordered pairs are also reversed). This should lead to a class discussion of what an inverse is, the process of finding an inverse, and the domain-range relationship between functions and their inverses.
Bloom's Levels: Apply, Apply
Webb's DOK: 2, 3


## Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Use Function Carnival from Desmos Teacher Activities to have students put different parent functions together in a piecewise function. Students will also need to use the concepts of increasing and decreasing intervals to identify where to place maximum and minimum points.

Summary of Engaging Learning Experiences for Topics

| Topic | Engaging Experience Title | Description | Suggested <br> Length of Time |
| :---: | :---: | :---: | :---: |
| Functions and Graphs | Just Move it Desmos | Students will complete the Move It Transformations activity on Desmos. | 45 minutes |
| Analyzing <br> Functions | Cell Phone Activity | After being introduced to the idea of a piecewise function students will be given a cell phone scenario with 2-3 different plans (for example plan A: $\$ 30$ monthly fee, 450 anytime minutes, $\$ .45$ for each additional minute, plan B: $\$ 45$ monthly fee, 900 anytime minutes, $\$ .30$ for each additional minute). Students will be given a task to create a piecewise function to represent each scenario, then answer questions about which is the best plan if you use 400 minutes each month, 475,600 , etc., verifying this both graphically and algebraically. | 30 minutes |
| Combining Functions | Chain Reaction | Students will be divided into groups of 4 or 5 and labeled with a number 1-5 (for groups of 4 one person will go twice). The teacher will prepare 5 functions (using a variety of ones discussed quadratic, linear, square root, etc.) and print these same 5 functions on note cards for each of the groups in the class. Each group will receive the set of 5 functions and distribute them to their group members in random order. The teacher will then call out a number, for example -1 . The 1 student in each group | 15 minutes |


|  |  | will evaluate his/her function at that value, the 2 students will then evaluate his/her function at the answer to student 1's function. This will continue on until all 5 functions have been evaluated. Answers will be checked, and students will then have an opportunity to go back and fix their chain if they were wrong. This is a good lead into discussing the composition of functions. |  |
| :---: | :---: | :---: | :---: |

## Unit 9: Polynomials and Rationals

Subject: College Algebra
Grade: 10, 11, 12
Name of Unit: Polynomials and Rationals
Length of Unit: 10 class periods
Overview of Unit: Students will graph polynomials and rational functions using domain, end behavior, and intercepts. Students will find all zeros of polynomial including complex zeros and multiplicity of zeros.

## Priority Standards for unit:

- Alg2.NQ.B.3: Know and apply the Fundamental Theorem of Algebra.
- Alg2.APR.A.5: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to sketch the function defined by the polynomial.
- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.


## Supporting Standards for unit:

- Alg2.APR.A.1: Extend the knowledge of factoring to include factors with complex coefficients.
- Alg2.APR.A.2: Understand the Remainder Theorem and use it to solve problems.
- Alg2.FM.A.1: Create functions and use them to solve applications of quadratic and exponential function model problems.
- NMP.FF. 1 Conceptualize quantities and define variables that are present in a given situation.
- NMP.FF. 2 Use multiple representations of functions to interpret and describe how two quantities change together.
- NMP.FF. 3 Measure, compute, describe, and interpret rates of change of quantities embedded in multiple representations.
- ISTE-COMPUTATIONAL THINKER.5.A - formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.
- ISTE-KNOWLEDGE COLLECTOR.3.A - plan and employ effective research strategies to locate information and other resources for their intellectual or creative pursuits.

| Unwrapped Concepts <br> (Students need to know) | Unwrapped Skills <br> (Students need to <br> be able to do) | Bloom's <br> Taxonomy <br> Levels | Webb's <br> DOK |
| :---: | :---: | :---: | :---: |
| the Fundamental Theorem of Algebra | Know | Remember | 1 |
| the Fundamental Theorem of Algebra | Apply | Apply | 3 |


| zeros of polynomials when suitable <br> factorizations are available | Identify | Understand | 1 |
| :---: | :---: | :---: | :---: |
| the zeros to sketch the function defined by the <br> polynomial. | Use | Understand | 2 |
| key characteristics of functions represented <br> graphically, with tables and with algebraic <br> symbolism to solve problems. | Identify | Understand | 2 |
| key characteristics of functions represented <br> graphically, with tables and with algebraic <br> symbolism to solve problems. | Interpret | Analyze | 3 |

## Essential Questions:

1. How can the number of solutions to a polynomial be determined?
2. How can the end behavior of a polynomial be determined?
3. How do domain issues affect the graph of a rational function?
4. How can the end behavior of a rational function be determined?

## Enduring Understanding/Big Ideas:

1. The degree of the polynomial determines the number of the solutions.
2. The degree and Leading Coefficient determine the end behavior.
3. Values not in the domain are either a hole or a vertical asymptote.
4. The end behavior will be determined by dividing quotient.

## Unit Vocabulary:

| Academic Cross-Curricular Words | Content/Domain Specific |
| :---: | :---: |
| Maximum | End behavior |
| Minimum | Zero |
| Domain | Polynomial |
| Range | Rational |
| Quotient | Synthetic Division |
| Remainder | Degree |
| Joint Variation | Leading Coefficient |
| Inverse Variation | Asymptote |
| Direct Variation | Quadratic |
| Proportional | Complex Number |
| Constant | Roots |
|  | Multiplicity |
|  | Intercepts |
|  | Bounds |
|  | Conjugate |

Resources for Vocabulary Development: textbook

## Topic 1: Polynomial Functions

## Engaging Experience 1

Title: Real World Extrema Problems
Suggested Length of Time: $1 / 2$ of a class period Standards Addressed

Priority:

- Alg2.APR.A.5: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to sketch the function defined by the polynomial.
- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.


## Supporting:

- Alg2.FM.A.1: Create functions and use them to solve applications of quadratic and exponential function model problems.
- NMP.FF. 1 Conceptualize quantities and define variables that are present in a given situation.
Detailed Description/Instructions: Students will be given different scenarios that require them to write a quadratic function to represent the problem. They will then use the equation to determine the optimal solution (max profit, min fencing, etc...)
Bloom's Levels: Identify, Use, Analyze
Webb's DOK: 1, 2, 3
Rubric: To be created


## Engaging Experience 2

Title: Analyze the graphs of polynomials
Suggested Length of Time: 1 class period Standards Addressed

## Priority:

- Alg2.APR.A.5: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to sketch the function defined by the polynomial.
- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.
Supporting:
- ISTE-COMPUTATIONAL THINKER.5.A - formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.

Detailed Description/Instructions: Students will use end behavior and multiplicity of zeros to graph several polynomials. The students then will use technology (graphing calculator or Desmos) to verify their answers.
Bloom's Levels: Identify, Use, Analyze
Webb's DOK: 1, 2, 3

## Engaging Experience 3

Title: Finding Zeros of Polynomials
Suggested Length of Time: 2 class periods

## Standards Addressed

Priority:

- Alg2.IF.A.1:
- Alg2.NQ.B.3: Know and apply the Fundamental Theorem of Algebra.
- Alg2.APR.A.5: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to sketch the function defined by the polynomial.


## Supporting:

- Alg2.APR.A.1: Extend the knowledge of factoring to include factors with complex coefficients.
- Alg2.APR.A.2: Understand the Remainder Theorem and use it to solve problems.
- ISTE-KNOWLEDGE COLLECTOR.3.A - plan and employ effective research strategies to locate information and other resources for their intellectual or creative pursuits.
Detailed Description/Instructions: Students will be given a variety of polynomial functions and be asked to use technology and theorems (possible rational zeros, Descartes' Rule of signs, and bounds) to determine potential zeros of the function. They will then use synthetic or polynomial division to reduce the function and identify zeros. Some zeros will be complex.
Bloom's Levels: Analyze
Webb's DOK: 3


## Topic 2: Rational Functions

## Engaging Experience 1

Title: Analyze Rational Functions
Suggested Length of Time: 3/4 of a class period

## Standards Addressed

Priority:

- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.
Supporting:
- ISTE-COMPUTATIONAL THINKER.5.A - formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.
Detailed Description/Instructions: Students will be tasked with taking a variety of rational functions and analyzing them to find domain, range, holes, asymptotes, end behavior, intercepts, and extrema. They will then create graphs of these functions both by hand and with the assistance of technology.
Bloom's Levels: Analyze
Webb's DOK: 3


## Engaging Experience 2

Title: Variation
Suggested Length of Time: 1/2 of a class period Standards Addressed

Priority:

- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.
Supporting:
- NMP.FF. 2 Use multiple representations of functions to interpret and describe how two quantities change together.
- NMP.FF. 3 Measure, compute, describe, and interpret rates of change of quantities embedded in multiple representations.
Detailed Description/Instructions: Students will work on getting a better understanding of the relationship between variable. Students will be given a scenario, and will find an equation and determine future data points using that equation. Potential scenarios could include: applying gas laws, or the gravitational force between two masses, or scenarios involving movement and examining acceleration or deceleration.
Bloom's Levels: Analyze
Webb's DOK: 3


## Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will be broken into groups and given sheets of paper of various sizes. They will construct an open topped box of maximum volume. Their product will be a physical box, the equation modeling their boxes volume, a screenshot of technology showing values of the maximum, and a justification of their findings.
The students then will find the two values they could have cut out to make a box of half the volume. If their solution can be found without technology, they must show the work by hand.

## Summary of Engaging Learning Experiences for Topics

| Topic | Engaging <br> Experience <br> Title | Description | Suggested <br> Length of <br> Time |
| :---: | :---: | :---: | :---: |
| Polynomial <br> Functions | Real World <br> Extrema <br> Problems | Students will be given different scenarios that <br> require them to write a quadratic function to <br> represent the problem. They will then use the <br> equation to determine the optimal solution (max <br> profit, min fencing, etc...) | $1 / 2$ of a <br> class <br> period |
| Polynomial <br> Functions | Analyze the <br> graphs of <br> polynomials | Students will use end behavior and multiplicity of <br> zeros to graph several polynomials. The students <br> then will use technology (graphing calculator or <br> Desmos) to verify their answers. | 1 class <br> period |
| Polynomial <br> Functions | Finding <br> Zeros of <br> Polynomials | Students will be given a variety of polynomial <br> functions and be asked to use technology and <br> theorems (possible rational zeros, Descartes' Rule of <br> signs, and bounds) to determine potential zeros of <br> the function. They will then use synthetic or | 2 class <br> periods |
| Rational <br> Functions | Analyze <br> Rational <br> Functions | Students will be tasked with taking a variety of <br> rational functions and analyzing them to find <br> domain, range, holes, asymptotes, end behavior, <br> intercepts, and extrema. They will then create graphs <br> of these functions both by hand and with the <br> assistance of technology. | $3 / 4$ of a <br> class <br> period |
| Rational <br> Functions | Variation | Students will work on getting a better understanding <br> of the relationship between variable. Students will be <br> given a scenario and will find an equation and <br> determine future data points using that equation. <br> Potential scenarios could include applying gas laws, <br> or the gravitational force between two masses, or <br> scenarios involving movement and examining <br> acceleration or deceleration. | $1 / 2$ of a <br> class <br> period |

## Unit 10: Exponentials and Logarithms

Subject: College Algebra
Grade: 10, 11, 12
Name of Unit: Exponentials and Logarithms
Length of Unit: 9 class periods
Overview of Unit: Students will graph exponential and logarithmic functions using characteristics of each function. Students will solve exponential and logarithmic equations.
Students will apply exponential and logarithmic functions to solve real world applications.

## Priority Standards for unit:

- Alg2.SSE.A.1: Develop the definition of logarithms based on properties of exponents.
- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.


## Supporting Standards for unit:

- Alg2.SSE.A.2: Use the inverse relationship between exponents and logarithms to solve exponential and logarithmic equations.
- Alg2.SSE.A.3: Use properties of logarithms to solve equations or find equivalent expressions.
- Alg2.SSE.A.4: Understand why logarithmic scales are used, and use them to solve problems.
- Alg2.IF.A.2: Translate between equivalent forms of functions.
- Alg2.BF.A.3: Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for linear, quadratic, cubic, square and cube root, absolute value, exponential and logarithmic functions.
- Alg2.FM.A.1: Create functions and use them to solve applications of quadratic and exponential function model problems.
- NMP.FF. 2 Use multiple representations of functions to interpret and describe how two quantities change together.
- ISTE-COMPUTATIONAL THINKER.5.B - collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problemsolving and decision-making.
- ISTE-KNOWLEDGE COLLECTOR.3.A - plan and employ effective research strategies to locate information and other resources for their intellectual or creative pursuits.

| Unwrapped Concepts <br> (Students need to know) | Unwrapped Skills <br> (Students need to be able to do) | Bloom's <br> Taxonomy <br> Levels | Webb's <br> DOK |
| :---: | :---: | :---: | :---: |
| the definition of logarithms based <br> on properties of exponents | Develop | Understand | 4 |
| key characteristics of functions <br> represented graphically, with <br> tables and with algebraic <br> symbolism to solve problems. | Identify | Understand | 2 |
| key characteristics of functions <br> represented graphically, with <br> tables and with algebraic | Interpret | Analyze | 3 |

## Essential Questions:

1. Why do exponential functions have a horizontal asymptote?
2. How are exponential and logarithmic functions related?
3. How do we evaluate a logarithmic function? (What is a logarithm?)
4. How do we apply exponential functions to the real world?
5. How do we apply logarithmic functions to the real world?

## Enduring Understanding/Big Ideas:

1. As the exponent gets closer to negative infinity, the value gets closer to 0 .
2. Exponential and logarithmic functions are inverse functions.
3. You rewrite the logarithm as an exponential and find the find the exponent. (It is an exponent.)
4. They are used to model exponential growth and compound interest.
5. They are used to solve exponential growth and compound interest problems. They are also used to model intensity of sound, earthquakes, and acidity of liquids.

Unit Vocabulary:

| Academic Cross-Curricular Words | Content/Domain Specific |
| :---: | :---: |
| Exponential | Logarithmic |
| Growth | Asymptote |
| Decay | Intercepts |
| Domain | Property |
| Range | Transformations |
| Constant | Laws |
|  | Base |


|  | Exponent |
| :---: | :---: |
|  | Initial Value |
| Population |  |
| Difference |  |

[^0]
## Topic 1: Exponential Functions

## Engaging Experience 1

Title: Graphing Exponential Functions
Suggested Length of Time: 1/2 of a class period Standards Addressed

Priority:

- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.


## Supporting:

- Alg2.BF.A.3: Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for linear, quadratic, cubic, square and cube root, absolute value, exponential and logarithmic functions.
Detailed Description/Instructions: Students will be giving several exponential functions of any base including $e$ to graph without the aid of a graphing utility. Students will graph these functions by using transformations and key characteristics common to all exponential functions such as domain, range, the horizontal asymptote, and key points.
Bloom's Levels: Understand
Webb's DOK: 2


## Topic 2: Logarithmic Functions

## Engaging Experience 1

Title: Develop an Understanding of Logarithms
Suggested Length of Time: $1 / 2$ of a class period Standards Addressed

Priority:

- Alg2.SSE.A.1: Develop the definition of logarithms based on properties of exponents.


## Supporting:

- Alg2.SSE.A.2: Use the inverse relationship between exponents and logarithms to solve exponential and logarithmic equations.
- Alg2.IF.A.2: Translate between equivalent forms of functions.
- NMP.FF. 2 Use multiple representations of functions to interpret and describe how two quantities change together.
- Alg2.SSE.A.3: Use properties of logarithms to solve equations or find equivalent expressions.
Detailed Description/Instructions: Students will convert logarithmic expressions into exponential functions with an unknown exponent. The students will then use the properties of exponents to solve for the missing exponent.
Bloom's Levels: Understand
Webb's DOK: 4


## Engaging Experience 2

Title: Graph Logarithms
Suggested Length of Time: $1 / 2$ of a class period
Standards Addressed
Priority:

- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.


## Supporting:

- Alg2.BF.A.3: Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for linear, quadratic, cubic, square and cube root, absolute value, exponential and logarithmic functions.

Detailed Description/Instructions: Students will use the graph of an exponential functions and the properties and symmetry of inverse functions to develop the graphs of logarithms of any base and state key characteristics such as domain, range, vertical asymptote, and key points. The students then will be asked to graph several logarithmic functions using transformations and then compare their graphs to a graphing utility and note the limitations of the graphing utility.
Bloom's Levels: Understand
Webb's DOK: 2

## Engaging Experience 3

Title: Use Properties of Logarithms
Suggested Length of Time: $1 / 2$ of a class period Standards Addressed

Priority:

- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.


## Supporting:

- Alg2.BF.A.3: Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for linear, quadratic, cubic, square and cube root, absolute value, exponential and logarithmic functions.
- ISTE-COMPUTATIONAL THINKER.5.B - collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.
Detailed Description/Instructions: Students will use the properties of logarithms to expand and condense expressions. To verify their work, students will be given functions and will graph the original functions and the expanded or condensed expression in the same window of a graphing utility to verify the functions are equivalent.
Bloom's Levels: Understand
Webb's DOK: 2


## Topic 3: Solve and Apply

## Engaging Experience 1

Title: Solving Exponential and Logarithmic Equations using a variety of methods
Suggested Length of Time: 1 class period Standards Addressed

Priority:

- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.
Supporting:
- Alg2.SSE.A.2: Use the inverse relationship between exponents and logarithms to solve exponential and logarithmic equations.
- Alg2.SSE.A.3: Use properties of logarithms to solve equations or find equivalent expressions.
Detailed Description/Instructions: Students will be given exponential and logarithmic functions in a variety of formats. They will be asked to use their knowledge of the inverse relationship of the functions and exponentials and logarithmic properties to solve those equations.
Bloom's Levels: Apply
Webb's DOK: 3


## Engaging Experience 2

Title: Exponential Growth and Decay
Suggested Length of Time: $3 / 4$ of a class period
Standards Addressed
Priority:

- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.
Supporting:
- Alg2.SSE.A.3: Use properties of logarithms to solve equations or find equivalent expressions.
- Alg2.SSE.A.4: Understand why logarithmic scales are used, and use them to solve problems.
- Alg2.FM.A.1: Create functions and use them to solve applications of quadratic and exponential function model problems.

Detailed Description/Instructions: Students will solve a variety of exponential growth applications. Population growth, radioactive decay, continuously compounded interest, and Newton's Law of Cooling will be included.
Bloom's Levels: Interpret
Webb's DOK: 3

## Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will do research to find the retail price of a car 5 years ago and then find the current trade in value. The students will use the data to find the rate of depreciation of the car annually and monthly.
Students then research to find the retail price of a brand-new car. They will then have to find a rate they will be given for a loan to buy that car. Finally, the students will calculate the accumulated amount they will owe if they had not made any payments for 5 years.

## Summary of Engaging Learning Experiences for Topics

| Topic | Engaging Experience Title | Description | Suggested <br> Length of Time |
| :---: | :---: | :---: | :---: |
| Exponential Functions | Graphing <br> Exponential <br> Functions | Students will be giving several exponential functions of any base including $e$ to graph without the aid of a graphing utility. Students will graph these functions by using transformations and key characteristics common to all exponential functions such as domain, range, the horizontal asymptote, and key points. | $1 / 2$ of a class period |
| Logarithmic Functions | Develop an Understanding of Logarithms | Students will convert logarithmic expressions into exponential functions with an unknown exponent. The students will then use the properties of exponents to solve for the missing exponent. | $1 / 2$ of a class period |
| Logarithmic Functions | Graph Logarithms | Students will use the graph of an exponential functions and the properties and symmetry of inverse functions to develop the graphs of logarithms of any base and state key characteristics such as domain, range, vertical asymptote, and key points. The students then will be asked to graph several logarithmic functions using transformations and then compare their graphs to a graphing utility and note the limitations of the graphing utility. | $1 / 2$ of a class period |
| Logarithmic Functions | Use Properties of Logarithms | Students will use the properties of logarithms to expand and condense expressions. To verify their work, students will be given functions and will graph the original functions and the expanded or condensed expression in the same window of a graphing utility to verify the functions are equivalent. | $1 / 2$ of a class period |


| Solve and <br> Apply | Solving <br> Exponential <br> and <br> Logarithmic <br> Equations <br> using a variety <br> methods | Students will be given exponential and <br> logarithmic functions in a variety of formats. <br> They will be asked to use their knowledge of <br> the inverse relationship of the functions and <br> exponentials and logarithmic properties to <br> solve those equations. | 1 class <br> period |
| :---: | :---: | :---: | :---: |
| Solve and <br> Apply | Exponential <br> Growth and <br> Decay | Students will solve a variety of exponential <br> growth applications. Population growth, <br> radioactive decay, continuously compounded <br> interest, and Newton's Law of Cooling will be <br> included. | $3 / 4$ of a class <br> period |

## Unit 11: Conics and Systems of Equations

Subject: College Algebra
Grade: 10, 11, 12
Name of Unit: Conics and Systems of Equations
Length of Unit: 10 class periods
Overview of Unit: Students will graph each conic section using its characteristics. Students will solve systems of equations including nonlinear systems. Students will apply systems of equations and inequalities to solve real world applications including linear programming.

## Priority Standards for unit:

- Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.
- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.


## Supporting Standards for unit:

- Alg2.IF.A.2: Translate between equivalent forms of functions.
- AL.9-12.F-CS. 1 Create graphs of conic sections, including parabolas, hyperbolas, ellipses, circles, and degenerate conics, from second-degree equations.
- AL.9-12.F-CS.1.a Formulate equations of conic sections from their determining characteristics.
- ISTE-COMPUTATIONAL THINKER.5.A - formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.
- Write equations of parabolas in vertex form (PC-P.2)
- Write equations of circles in standard form (PC-P.5)
- Write equations of ellipses in standard form (PC-P.9)
- Write equations of hyperbolas in standard form (PC-P.12)
- Convert equations of conic sections from general to standard form (PC-P.13)

| Unwrapped Concepts <br> (Students need to know) | Unwrapped Skills <br> (Students need to be able to do) | Bloom's <br> Taxonomy <br> Levels | Webb' <br> s DOK |
| :---: | :---: | :---: | :---: |
| systems of equations that may <br> include non-linear equations and <br> inequalities. | Create |  |  |
| systems of equations that may <br> include non-linear equations and <br> inequalities. | Solve | Create | 3 |


| key characteristics of functions <br> represented graphically, with <br> tables and with algebraic <br> symbolism to solve problems. | Identify |  |  |
| :---: | :---: | :---: | :---: |
| key characteristics of functions <br> represented graphically, with <br> tables and with algebraic |  | Understand | 2 |
| symbolism to solve problems. | Interpret | Analyze | 3 |

## Essential Questions:

1. Why is it important to be able to graph a conic section?
2. How solve a real-world problem that has several variables?
3. Why is it important to know several methods to solve systems of equations?
4. How do I write the solution to a system that has infinite many answers?
5. How can systems of inequalities be used to find an optimal solution?

## Enduring Understanding/Big Ideas:

1. The graphs can be used to solve nonlinear systems.
2. Write a system that has as many equations as there are variables.
3. Some systems can only be solved graphically or only by substitution. Elimination is the only method that is used when we apply systems to matrices.
4. Write the answer using set-builder notation or parametrics.
5. They can be used in Linear Programming, to find what needs to be done to find the maximum and minimum values.

## Unit Vocabulary:

| Academic Cross-Curricular Words | Content/Domain Specific |
| :---: | :---: |
| Circle <br> Nonlinear | Parabola Ellipse Hyperbola Vertex (Vertices) Focus (Foci) Directrix Axis (Major / Minor) Focal Diameter Asymptote System Consistent Inconsistent |


|  | Independent |
| :---: | :---: |
| Dependent |  |
| Parametric |  |
| Set-Builder |  |
| Extrema |  |
| Optimization |  |
| Feasible Region |  |
| Objective Function |  |

Resources for Vocabulary Development: Textbook

## Topic 1: Conic Sections

## Engaging Experience 1

Title: Conic Sections Centered about the Origin with the white boards
Suggested Length of Time: 1-1/2 class periods (Parabolas-1/2 class period, Circles/Ellipses- $1 / 2$ class period, Hyperbolas- $1 / 2$ class period)
Standards Addressed

## Priority:

- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.


## Supporting:

- Alg2.IF.A.2: Translate between equivalent forms of functions.
- ISTE-COMPUTATIONAL THINKER.5.A - formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.
- AL.9-12.F-CS. 1 Create graphs of conic sections, including parabolas, hyperbolas, ellipses, circles, and degenerate conics, from second-degree Equations.
- AL.9-12.F-CS.1.a Formulate equations of conic sections from their determining characteristics.
Detailed Description/Instructions: Students will be given equations of conic sections in standard form and will then sketch their graphs and identity their key features. Students will be given the graph of a conic and then write its equation in standard form. When given the key features of a conic, students will write the equation and graph. They can use technology to verify their answers.
Bloom's Levels: Understand
Webb's DOK: 2


## Engaging Experience 2

Title: Transforming Conic Sections
Suggested Length of Time: 1-1/2 class periods (Parabolas- $1 / 2$ class period, Circles/Ellipses- $1 / 2$ class period, Hyperbolas- $1 / 2$ class period). This experience will be done concurrently with engaging experience 1.

## Standards Addressed

Priority:

- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.


## Supporting:

- Alg2.IF.A.2: Translate between equivalent forms of functions.
- AL.9-12.F-CS. 1 Create graphs of conic sections, including parabolas, hyperbolas, ellipses, circles, and degenerate conics, from second-degree Equations.
- AL.9-12.F-CS.1.a Formulate equations of conic sections from their determining characteristics.
Detailed Description/Instructions: Students will be given translated conic sections in general and standard form and tasked with graphing those sections. If the section is in general form they will need to convert it is standard form by completing the square.
Bloom's Levels: Analyze
Webb's DOK: 3


## Topic 2: Systems of Equations

## Engaging Experience 1

Title: Writing dependent systems in parametric form
Suggested Length of Time: 1/2 of a class period

## Standards Addressed

Priority:

- Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.
Supporting:
- Alg2.IF.A.2: Translate between equivalent forms of functions.

Detailed Description/Instructions: Students will solve linear systems and 3 variables that have infinite solutions. They will have to decided which type of dependent system they have been given and write the answer in the appropriate form, i.e. set-builder notation or parametric form.
Bloom's Levels: Apply
Webb's DOK: 2

## Engaging Experience 2

Title: Extending Methods of solving Linear Systems to Nonlinear on White Boards
Suggested Length of Time: 15 minutes
Standards Addressed
Priority:

- Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.
Detailed Description/Instructions: The student will apply substitution, elimination, and graphing to systems involving nonlinear equations.
Bloom's Levels: Apply
Webb's DOK: 2


## Engaging Experience 3

Title: Graphically solving systems of inequalities and applying to real world situations
Suggested Length of Time: 1 class period
Standards Addressed
Priority:

- Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.
Detailed Description/Instructions: Students will be presented with systems of inequalities involving linear and nonlinear inequalities and then use the process in linear programming.
Bloom's Levels: Apply
Webb's DOK: 2


## Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will do the linear programing extension at the end of chapter 5 in the textbook to find optimal solutions. Students will need to show the system of linear inequalities and the vertices of the feasible region.

## Summary of Engaging Learning Experiences for Topics

| Topic | Engaging <br> Experience <br> Title | Description | Suggested Length of Time |
| :---: | :---: | :---: | :---: |
| Conic <br> Sections | Conic Sections Centered about the Origin with the white boards | Students will be given equations of conic sections in standard form and will then sketch their graphs and identity their key features. Students will be given the graph of a conic and then write its equation in standard form. When given the key features of a conic, students will write the equation and graph. They can use technology to verify their answers. | $1-1 / 2$ class periods (Parabolas- $1 / 2$ class period, Circles/Ellipses- $1 / 2$ class period, Hyperbolas- $1 / 2$ class period) |
| Conic <br> Sections | Transforming Conic Sections | Students will be given translated conic sections in general and standard form and tasked with graphing those sections. If the section is in general form they will need to convert it is standard form by completing the square. | $1-1 / 2$ class periods (Parabolas- $1 / 2$ class period, Circles/Ellipses- $1 / 2$ class period, Hyperbolas- $1 / 2$ class period). This experience will be done concurrently with engaging experience 1 |
| Systems <br> of <br> Equations | Writing dependent systems in parametric form | Students will solve linear systems and 3 variables that have infinite solutions. They will have to decided which type of dependent system they have been given and write the answer in the appropriate form, i.e. set-builder notation or parametric form. | $1 / 2$ of a class period |
| Systems <br> of <br> Equations | Extending <br> Methods of solving Linear | The student will apply substitution, elimination, and graphing to systems involving nonlinear equations. | 15 minutes |


|  | Systems to <br> Nonlinear on <br> White Boards |  |  |
| :---: | :---: | :---: | :---: |
| Systems <br> of <br> Equations | Graphically <br> solving systems <br> of inequalities <br> and applying to <br> real world <br> situations | Students will be presented with systems of <br> inequalities involving linear and nonlinear <br> inequalities and then use the process in <br> linear programming. | 1 class period |

## Unit 12: Systems and Matrices

Subject: College Algebra
Grade: 10, 11, 12
Name of Unit: Systems and Matrices
Length of Unit: 7 class periods
Overview of Unit: Students will learn basic matrix operations to evaluate expressions and solve real world applications. Students will use matrices to solve linear systems including augmented matrices, inverse matrices, and Cramer's Rule.

## Priority Standards for unit:

- Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.


## Supporting Standards for unit:

- N 406. Add two matrices that have whole number entries
- N 607. Use relations involving addition, subtraction, and scalar multiplication of vectors and of matrices
- N 706. Apply properties of matrices and properties of matrices as a number system
- ISTE-KNOWLEDGE COLLECTOR.3.D - build knowledge by actively exploring realworld issues and problems, developing ideas and theories and pursuing answers and solutions.
- ISTE-COMPUTATIONAL THINKER.5.A - formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.

| Unwrapped Concepts <br> (Students need to know) | Unwrapped Skills <br> (Students need to be <br> able to do) | Bloom's <br> Taxonomy <br> Levels | Webb's <br> DOK |
| :---: | :---: | :---: | :---: |
| systems of equations that may include non- <br> linear equations and inequalities. | Create | Create | 3 |
| systems of equations that may include non- <br> linear equations and inequalities. | Solve | Apply | 2 |

## Essential Questions:

1. How are matrices related to systems of linear equations?
2. Why are learning matrix solutions to systems beneficial?
3. How do I find and evaluate the determinant of a square matrix larger than a $2 \times 2$ ?

## Enduring Understanding/Big Ideas:

1. Linear systems can be rewritten as an augmented matrix or a matrix equation.
2. Technology can be used to solve linear systems extremely quickly when written as matrix.
3. By using expansion by minors, a square can be broken into several $2 \times 2$ matrices.

## Unit Vocabulary:

| Academic Cross-Curricular Words | Content/Domain Specific |
| :---: | :---: |
|  | Augmented |
|  | Inverse |
|  | Row Echelon |
| Gaussian |  |
| Rows |  |
|  | Columns |
| Scalar |  |
|  | Order (Dimensions) |
| Element |  |
|  | Determinant |
| Minor |  |
|  | Cofactor |

Resources for Vocabulary Development: Textbook

## Topic 1: Matrices

## Engaging Experience 1

Title: Real-World Problem with Matrix Operations
Suggested Length of Time: 15 minutes

## Standards Addressed

Priority:

- Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.


## Supporting:

- N 406. Add two matrices that have whole number entries
- N 607. Use relations involving addition, subtraction, and scalar multiplication of vectors and of matrices
- ISTE-KNOWLEDGE COLLECTOR.3.D - build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.
Detailed Description/Instructions: Students will model real-world data using a system of equations and will perform the operations of addition, subtraction, and scalar multiplication. The website https://meangreenmath.files.wordpress.com/2016/01/spacemath.png contains a sample problem. Afterwards, students will model the same process using Microsoft Excel in order to make a connection between an abstract concept in mathematics and a concrete program that students use on a weekly basis.
Bloom's Levels: Create
Webb's DOK: 3


## Engaging Experience 2

Title: Using Augmented Matrices to solve linear systems
Suggested Length of Time: 1 class period
Standards Addressed
Priority:

- Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.


## Supporting:

- N 706. Apply properties of matrices and properties of matrices as a number system
- ISTE-COMPUTATIONAL THINKER.5.A - formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.


## Detailed Description/Instructions:

Students will rewrite linear systems of equations as an augmented matrix and solve them using Gaussian and Gauss-Jordan Elimination. Students then will verify their answers by using a graphing calculator to put the matrix in Reduced Row Echelon Form.
Bloom's Levels: Apply
Webb's DOK: 2

## Engaging Experience 3

Title: Using a Matrix Equation to solve linear systems
Suggested Length of Time: 2 class periods
Standards Addressed
Priority:

- Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.


## Supporting:

- N 706. Apply properties of matrices and properties of matrices as a number system
- ISTE-COMPUTATIONAL THINKER.5.A - formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.
Detailed Description/Instructions:
Students will rewrite linear systems of equations as a matrix equation and solve them using an Inverse Matrix and Cramer's Rule. Students will use a graphing utility to find the inverse and determinants of larger systems.
Bloom's Levels: Apply
Webb's DOK: 2


## Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will be handed a bag consisting of 4 types of coins and be told the total amount of money and number of coins. They students will also be given two pieces of information that will relate the amount of the different types of coins. Students will use this information to determine how many of each type of coin is in the bag. Students will need to produce the systems of equations and matrices but may use technology to solve. To confirm their results, the students will then open the bag and count the coins.

## Summary of Engaging Learning Experiences for Topics

| Topic | Engaging <br> Experience Title | Description | Suggested <br> Length of <br> Time |
| :---: | :---: | :---: | :---: |
| Matrices | Real-World <br> Problem with <br> Matrix Operations | Students will model real-world data using a <br> system of equations and will perform the <br> operations of addition, subtraction, and scalar <br> multiplication. The website <br> https://meangreenmath.files.wordpress.com/2016 | 15 minutes |
| Matrices | Using Augmented <br> Matrices to solve <br> linear systems | Students will rewrite linear systems of equations <br> as an augmented matrix and solve them using <br> Afterwards, students will model the same <br> process using Microsoft Excel in order to make a <br> connection between an abstract concept in <br> mathematics and a concrete program that <br> students use on a weekly basis. | 1 class <br> period |
| Matrices | Using a Matrix <br> Equation to solve <br> linear systems then will verify their answers by using <br> a graphing calculator to put the matrix in <br> Reduced Row Echelon Form. | Students will rewrite linear systems of equations <br> as a matrix equation and solve them using an <br> Inverse Matrix and Cramer's Rule. Students will <br> use a graphing utility to find the inverse and <br> determinants of larger systems. | periods <br> pers |

## Unit of Study Terminology

Appendices: All Appendices and supporting material can be found in this course's shell course in the District's Learning Management System.

Assessment Leveling Guide: A tool to use when writing assessments in order to maintain the appropriate level of rigor that matches the standard.

Big Ideas/Enduring Understandings: Foundational understandings teachers want students to be able to discover and state in their own words by the end of the unit of study. These are answers to the essential questions.

Engaging Experience: Each topic is broken into a list of engaging experiences for students. These experiences are aligned to priority and supporting standards, thus stating what students should be able to do. An example of an engaging experience is provided in the description, but a teacher has the autonomy to substitute one of their own that aligns to the level of rigor stated in the standards.

Engaging Scenario: This is a culminating activity in which students are given a role, situation, challenge, audience, and a product or performance is specified. Each unit contains an example of an engaging scenario, but a teacher has the ability to substitute with the same intent in mind.

Essential Questions: Engaging, open-ended questions that teachers can use to engage students in the learning.

Priority Standards: What every student should know and be able to do. These were chosen because of their necessity for success in the next course, the state assessment, and life.

Supporting Standards: Additional standards that support the learning within the unit.
Topic: These are the main teaching points for the unit. Units can have anywhere from one topic to many, depending on the depth of the unit.

Unit of Study: Series of learning experiences/related assessments based on designated priority standards and related supporting standards.

Unit Vocabulary: Words students will encounter within the unit that are essential to understanding. Academic Cross-Curricular words (also called Tier 2 words) are those that can be found in multiple content areas, not just this one. Content/Domain Specific vocabulary words are those found specifically within the content.


[^0]:    Resources for Vocabulary Development: Textbook

