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## **High School College Algebra Curriculum**

**Course Description:** The class consists of two Metropolitan Community College courses: Math 110, Intermediate Algebra and Math 120, College Algebra for 6 hours college credit. The catalog description for Math 110 is a study of functions and their graphs, systems of linear equations, application problems, linear and quadratic inequalities, absolute value equations and inequalities, rational exponents, radicals, quadratic equations, ratios and proportions. The catalog description for Math 120 is a study of various types of equations and inequalities, functions and their inverses, theory of higher degree equations, systems of equations, determinants, logarithmic and exponential functions, conic sections, sequences and series, and the Binomial Theorem.

#### **Scope and Sequence:**

Timeframe	Unit	Instructional Topics
8 class periods	1 Variable Equations and Inequalities	Topic 1: Set Theory Topic 2: Properties of Real Numbers Topic 3: Linear Equations Topic 4: Inequalities
4 class periods	2 Variable Equations and Inequalities	Topic 1: Coordinate Geometry Topic 2: Linear Functions Topic 3: Functions
3 class periods	Systems	Topic 1: Solve and Apply
5 class periods	Exponents and Polynomials	Topic 1: Simplify Polynomials Topic 2: Factoring
6 class periods	Rationals	Topic 1: Simplify Rational Expressions Topic 2: Solve and Apply

6 class periods	Radicals	Topic 1: Simplifying Radicals Topic 2: Solving Radical Equations Topic 3: Complex Numbers
6 class periods	Quadratics	Topic 1: Polynomial Functions Topic 2: Solving and Applying
9 class periods	Functions	Topic 1: Functions and Graphs Topic 2: Analyzing Functions Topic 3: Combining Functions Topic 4: Inverse Functions
10 class periods	Polynomials and Rationals	Topic 1: Polynomial Functions Topic 2: Rational Functions
9 class periods	Exponentials and Logarithms	Topic 1: Exponential Functions Topic 2: Logarithmic Functions Topic 3: Solve and Apply
10 class periods	Conics and Systems of Equations	Topic 1: Conic Sections Topic 2: Systems of Equations

## Unit 1: 1 Variable Equations and Inequalities

### Subject: College Algebra

Grade: 10, 11, 12

Name of Unit: 1 Variable Equations and Inequalities

Length of Unit: 8 class periods

**Overview of Unit**: In this unit, students will solve and model with linear equations and inequalities, utilize the definitions of real numbers and their properties, and properly communicate solutions using the concepts of set theory.

### **Priority Standards for unit:**

• Alg2.REI.A.1: Create and solve equations and inequalities, including those that involve absolute value.

### Supporting Standards for unit:

• ISTE-KNOWLEDGE COLLECTOR.3.D - build knowledge by actively exploring realworld issues and problems, developing ideas and theories and pursuing answers and solutions.

Unwrapped Concepts	Unwrapped Skills	Bloom's	Webb's
(Students need to know)	(Students need to be able to do)	<b>Taxonomy Levels</b>	DOK
equations and inequalities,			
including those that involve			
absolute value.	Create	Create	3
equations and inequalities,			
including those that involve			
absolute value.	Solve	Apply	2

### **Essential Questions:**

- 1. How do you use interval notation to communicate solutions?
- 2. How do you use the properties of real numbers to evaluate expressions and solve equations?
- 3. How is order of operations used to isolate a variable in an equation?
- 4. How do you solve compound inequalities?

### **Enduring Understanding/Big Ideas:**

- 1. Write solutions from the smallest endpoint to largest endpoint using parentheses or brackets correctly.
- 2. They are used to make computations easier and quicker than relying only on following the order of operations.

- 3. Variables are isolated by applying the order of operations backwards with inverses.
- 4. They are solved by determining if we have an and/or statement and communicating the solution using correct notation.

Academic Cross-Curricular Words	Content/Domain Specific
Evaluate	Inequality
Intersection	Absolute value
Union	Algebraic expression
	Numeric Expression
	Additive inverse
	Multiplicative Inverse
	Irrational
	Rational
	Integer
	Whole
	Natural
	Expression vs. Equation
	Interval Notation
	Sets
	Element
	Subset

### **Unit Vocabulary:**

Resources for Vocabulary Development: textbook

## Topic 1: Set Theory

**Engaging Experience 1** 

Title: Set Activity Suggested Length of Time: 10 minutes Standards Addressed

Priority:

• Alg2.REI.A.1: Create and solve equations and inequalities, including those that involve absolute value.

**Detailed Description/Instructions**: Teacher will call out common traits among students in the class to make a set of students in the class. Based on what instructions are called out, students will stand or sit, demonstrating concepts of subsets, elements, unions, and intersections.

Bloom's Levels: Create and Apply

**Webb's DOK:** 3, 2

## Topic 2: Properties of Real Numbers

**Engaging Experience 1** 

Title: Properties with Algebra Tiles

Suggested Length of Time: 15 minutes

**Standards Addressed** 

Priority:

• Alg2.REI.A.1: Create and solve equations and inequalities, including those that involve absolute value.

**Detailed Description/Instructions:** As this is the first time that students will have seen the distributive property using variables in College Algebra, we will distribute and divide variable expressions using algebra tiles. This will strengthen various properties and provide a lead-in to future lessons.

**Bloom's Levels:** Create and Apply **Webb's DOK:** 3, 2

### Topic 3: Linear Equations

**Engaging Experience 1** 

Title: Reverse Procedures to demonstrate isolation Suggested Length of Time: 10 minutes Standards Addressed

Priority:

• Alg2.REI.A.1: Create and solve equations and inequalities, including those that involve absolute value.

**Detailed Description/Instructions:** Students will pair up, one will write out a procedure (i.e., the process of putting a gas nozzle in a car), the other student will write the reverse procedure (i.e., taking the nozzle out). They will then apply this to a linear equation using the reverse order of operations.

**Bloom's Levels:** Create and Apply **Webb's DOK:** 3, 2

## Topic 4: Inequalities

### **Engaging Experience 1**

**Title:** Inequalities from a graphical/numerical perspective **Suggested Length of Time:** 10 minutes **Standards Addressed** 

Priority:

• Alg2.REI.A.1: Create and solve equations and inequalities, including those that involve absolute value.

**Detailed Description/Instructions:** Students will be given various problems that provide a more meaningful approach to solving inequalities. These examples will include types that have no solution, encompass all real numbers, solutions that contain a set and one of its subsets, and other "non-traditional" problems that lead to students thinking critically about the definitions of inequalities, and statements, and or statements.

Bloom's Levels: Create and Apply

**Webb's DOK:** 3, 2

## **Engaging Scenario**

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Mix It Up - In this activity, students will work with mixtures of two colors of beads to understand the effect of combining two different mixtures to predict the percent concentration of the final mixture.

# Summary of Engaging Learning Experiences for Topics

Торіс	Engaging Experience Title	Description	Suggested Length of Time
Set Theory	Set Activity	Teacher will call out common traits among students in the class to make a set of students in the class. Based on what instructions are called out, students will stand or sit, demonstrating concepts of subsets, elements, unions, and intersections.	10 minutes
Properties of Real Numbers	Properties With Algebra Tiles	As this is the first time that students will have seen the distributive property using variables in College Algebra, we will distribute and divide variable expressions using algebra tiles. This will strengthen various properties and provide a lead-in to future lessons.	15 minutes
Linear Equations	Reverse Procedures to demonstrate isolation	Students will pair up, one will write out a procedure (i.e., the process of putting a gas nozzle in a car), the other student will write the reverse procedure (i.e., taking the nozzle out). They will then apply this to a linear equation using the reverse order of operations.	10 minutes
Inequalities	Inequalities from a graphical/numerical perspective	Students will be given various problems that provide a more meaningful approach to solving inequalities. These examples will include types that have no solution, encompass all real numbers, solutions that contain a set and one of its subsets, and other "non-traditional" problems that lead to students thinking critically about the definitions of inequalities, and statements, and or statements.	10 minutes

## Unit 2: 2 Variable Equations and Inequalities

### Subject: College Algebra

Grade: 10, 11, 12

Name of Unit: 2 Variable Equations and Inequalities

Length of Unit: 4 class periods

**Overview of Unit:** In this unit students will graph and write linear equations. They will work with equations in standard form, point-slope form, and slope-intercept form. In addition, they will find equations that are parallel and perpendicular.

### **Priority Standards for unit:**

• Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

### Supporting Standards for unit:

- Alg2.IF.A.2: Translate between equivalent forms of functions.
- NMP.FF.2 Use multiple representations of functions to interpret and describe how two quantities change together.
- NMP.FF.3 Measure, compute, describe, and interpret rates of change of quantities embedded in multiple representations.
- ISTE-COMPUTATIONAL THINKER.5.B collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.

Unwrapped Concepts (Students	Unwrapped Skills	Bloom's Taxonomy	Webb'
need to know)	(Students need to be able to do)	Levels	s DOK
key characteristics of functions			
represented graphically, with			
tables and with algebraic			
symbolism to solve problems	Identify	Remember	1
key characteristics of functions			
represented graphically, with			
tables and with algebraic			
symbolism to solve problems	Interpret	Analyze	3

### **Essential Questions:**

- 1. How is the slope of two lines related to their graphs?
- 2. How do we use given information to write the equation or construct a graph of a linear equation?

- 3. How is the graph of a linear equation different than the graph of a linear inequality?
- 4. How are relations and functions similar and different?

### **Enduring Understanding/Big Ideas:**

- 1. Find the slope of parallel and perpendicular lines and understand the relationship between the two.
- 2. Write and graph linear equations in Point-Slope, Standard, and Slope-Intercept forms
- 3. Linear equations contain the set of all solutions to an equation while linear inequalities contain the set of all solutions that satisfy an inequality.
- 4. Functions are relations that have a unique *y* value for each *x*-value.

#### **Unit Vocabulary:**

Academic Cross-Curricular Words	<b>Content/Domain Specific</b>	
Variable	Linear	
Slope	Coordinate plane	
Intercept	Parallel	
Rate of Change	Perpendicular	
Scatterplot	Point-Slope Form	
Intersection	Slope-Intercept Form	
Equation	Standard Form	
Independent Variable	Inequality	
Dependent Variable	Function	
-	Relation	
	Reciprocal	

Resources for Vocabulary Development: textbook

## Topic 1: Coordinate Geometry

### **Engaging Experience 1**

**Title:** Deriving formulas for distance and midpoint using the Pythagorean Theorem **Suggested Length of Time:** 15 minutes

### **Standards Addressed**

Priority:

• Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

Supporting:

• Alg2.IF.A.2: Translate between equivalent forms of functions.

**Detailed Description/Instructions:** Instead of giving students the formulas they will need for distance and midpoint, we will prove them using the Pythagorean Theorem, which students remember from Geometry.

Bloom's Levels: Analyze Webb's DOK: 3

### Topic 2: Linear Functions

**Engaging Experience 1** 

**Title:** Writing Equations of Lines Line-Up **Suggested Length of Time**: 15 minutes **Standards Addressed** 

Priority:

• Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

Supporting:

• NMP.FF.2 Use multiple representations of functions to interpret and describe how two quantities change together.

**Detailed Description/Instructions:** Students will be given a card that has given information about their line. This might be a point and a slope, two points, or information about a parallel/perpendicular line. Students will find the equation of their line, written in slope-intercept form. Afterwards, they will line up from smallest *y*-intercept to largest *y*-intercept. Count the number of mistakes (if any) and have students complete the problem of the person standing next to them to check for accuracy.

Bloom's Levels: Analyze Webb's DOK: 3

## Topic 3: Functions

**Engaging Experience 1** 

Title: Marbleslides Lines in Desmos Suggested Length of Time: 20 minutes Standards Addressed

Priority:

• Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

Supporting:

- NMP.FF.3 Measure, compute, describe, and interpret rates of change of quantities embedded in multiple representations.
- ISTE-COMPUTATIONAL THINKER.5.B collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.

**Detailed Description/Instructions:** Students will follow prompts in a Desmos activity that allows them to construct lines that intersect various "stars." Students will need to create multiple functions to make the scenario work and also keep mindful about domain restrictions, although they do not need to be fluent with the concept of domain at this point in time.

**Bloom's Levels:** Analyze Webb's DOK: 3

### **Engaging Scenario**

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will complete a linear regression project. Students will analyze the linear relationship (correlation) between two variables of their choosing. The project consists of researching data, analysis, and presenting the data and analysis in a formal report.

# Summary of Engaging Learning Experiences for Topics

Торіс	Engaging Experience Title	Description	Suggested Length of Time
Coordinate Geometry	Deriving formulas for distance and midpoint using the Pythagorean Theorem	Instead of giving students the formulas they will need for distance and midpoint, we will prove them using the Pythagorean Theorem, which students remember from Geometry.	15 minutes
Linear Functions	Writing Equations of Lines Line-Up	Students will be given a card that has given information about their line. This might be a point and a slope, two points, or information about a parallel/perpendicular line. Students will find the equation of their line, written in slope-intercept form. Afterwards, they will line up from smallest y-intercept to largest y-intercept. Count the number of mistakes (if any) and have students complete the problem of the person standing next to them to check for accuracy.	15 minutes
Functions	Marbleslides Lines in Desmos	Students will follow prompts in a Desmos activity that allows them to construct lines that intersect various "stars." Students will need to create multiple functions to make the scenario work and also keep mindful about domain restrictions, although they do not need to be fluent with the concept of domain at this point in time.	20 minutes

## Unit 3: Systems

Subject: College Algebra

**Grade:** 10, 11, 12 **Name of Unit:** Systems

Length of Unit: 3 class periods

**Overview of Unit:** In this unit students will solve systems of linear equations in two and three variables.

### **Priority Standards for unit:**

- Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.
- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

### **Supporting Standards for unit:**

- ISTE-KNOWLEDGE COLLECTOR.3.D build knowledge by actively exploring realworld issues and problems, developing ideas and theories and pursuing answers and solutions.
- ISTE-COMPUTATIONAL THINKER.5.A formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.

Unumeral Concepts	Unumers of Shills	Bloom's	Wahhla
Unwrapped Concepts	Unwrapped Skills	Taxonomy	
(Students need to know)	(Students need to be able to do)	Levels	DOK
systems of equations that may			
include non-linear equations and			
inequalities.	Create	Create	3
systems of equations that may			
include non-linear equations and			
inequalities.	Solve	Apply	2
key characteristics of functions			
represented graphically, with			
tables and with algebraic			
symbolism to solve problems.	Identify	Understand	2
key characteristics of functions			
represented graphically, with			
tables and with algebraic			
symbolism to solve problems.	Interpret	Analyze	3

### **Essential Questions:**

- 1. How do we solve a linear system of two and three variables?
- 2. What are the solution types of a linear system, and how are they used to classify the system?

### **Enduring Understanding/Big Ideas:**

- 1. Use various strategies including graphing, substitution, and linear combination
- 2. The solutions can be independent and consistent, dependent and consistent, or inconsistent, and they are used to describe any linear system.

#### **Unit Vocabulary:**

Academic Cross-Curricular Words	Content/Domain Specific
	Substitution
	Linear Combination
	Consistent
	Inconsistent
	Independent
	Dependent
	Commutativity

**Resources for Vocabulary Development:** textbook

## Topic 1: Solve and Apply

#### **Engaging Experience 1**

Title: The Tortoise and Hare Activity Suggested Length of Time: 45 minutes Standards Addressed

Priority:

- Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.
- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

Supporting:

• ISTE-COMPUTATIONAL THINKER.5.A - formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.

**Detailed Description/Instructions:** The Tortoise and the Hare finally have their long-awaited rematch. The Tortoise gets a 1,000-foot lead and runs at 9 inches per second. The Hare begins at the starting line and runs at a rate of 6 feet per second. There is also a rat in this race. The Rat starts 1,200 feet ahead of the Hare and runs back towards the starting line at a rate of 2 feet per second.

In this problem, students extract data from a story in order to write, manipulate, and graph systems of equations. It offers students a context to understand the relationships among data, equations, graphs and solutions.

**Bloom's Levels:** Create and Apply **Webb's DOK:** 3, 2

### **Engaging Scenario**

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will research two cars, given specific guidelines. One is an older model, cheap, sports car that gets poor gas mileage, the other a newer more economical, more expensive vehicle. Students will research the cost and gas mileage. They will then model this information with linear equations, graph and determine at what point in time the cost of the two vehicles would intersect. For the final product they would then have to summarize by determining which car would be the best and give mathematically supported reasons.

# Summary of Engaging Learning Experiences for Topics

Торіс	Engaging Experience Title	Description	Suggested Length of Time
Solve and Apply	The Tortoise and Hare Activity	The Tortoise and the Hare finally have their long-awaited rematch. The Tortoise gets a 1000-foot lead and runs at 9 inches per second. The Hare begins at the starting line and runs at a rate of 6 feet per second. There is also a rat in this race. The Rat starts 1,200 feet ahead of the Hare and runs back towards the starting line at a rate of 2 feet per second. In this problem, students extract data from a story in order to write, manipulate, and graph systems of equations. It offers students a context to understand the relationships among data, equations, graphs and solutions.	45 minutes

## Unit 4: Exponents and Polynomials

Subject: College Algebra

**Grade:** 10, 11, 12

Name of Unit: Exponents and Polynomials

Length of Unit: 5 class periods

**Overview of Unit:** In this unit students will simplify expressions using the rules of exponents. Students will also factor and solve quadratic (and quadratic type) equations.

### **Priority Standards for unit:**

• Alg1.SSE.A.2: Analyze the structure of polynomials to create equivalent expressions or equations.

### **Supporting Standards for unit:**

- Alg2.FM.A.1: Create functions and use them to solve applications of quadratic and exponential function model problems.
- ISTE-COMPUTATIONAL THINKER.5.C break problems into component parts, extract key information, and develop descriptive models to understand complex systems or facilitate problem-solving.
- ISTE-GLOBAL COLLABORATOR.7.C contribute constructively to project teams, assuming various roles and responsibilities to work effectively toward a common goal.
- ISTE-INNOVATIVE DESIGNER.4.B select and use digital tools to plan and manage a design process that considers design constraints and calculated risks.

		Bloom's	
Unwrapped Concepts	Unwrapped Skills	Taxonomy	Webb's
(Students need to know)	(Students need to be able to do)	Levels	DOK
the structure of polynomials to			
create equivalent expressions or			
equations.	Analyze	Analyze	2

### **Essential Questions**:

- 1. How do you simplify expressions using exponents?
- 2. How do you write a polynomial in completely factored form?
- 3. What is the zero-factor property and how do you use it to solve polynomial equations?

### **Enduring Understanding/Big Ideas:**

- 1. Use the properties of exponents.
- 2. Use various strategies of factoring including: GCF, difference of squares, perfect square trinomials, trinomials, and sum/difference of cubes.

3. When the product of two real numbers is zero, at least one of them is zero. This can be used to solve for each factor.

### <u>Unit Vocabulary:</u>

Academic Cross-Curricular Words	<b>Content/Domain Specific</b>
Properties/rules/laws	Simplest form
Coefficient	Factor
	Polynomial
	Base
	Degree
	Monomial
	Binomial
	Trinomial
	Perfect Square Trinomial
	Greatest Common Factor
	FOIL
	Prime
	Zero-Product Property
Resources for Vocabulary Development: textbook	

## Topic 1: Simplify Polynomials

**Engaging Experience 1** 

Title: Polynomial Operation Mix-And-Match Activity Suggested Length of Time: 20 minutes Standards Addressed

Priority:

• Alg1.SSE.A.2: Analyze the structure of polynomials to create equivalent expressions or equations.

Supporting:

• Alg2.FM.A.1: Create functions and use them to solve applications of quadratic and exponential function model problems.

**Detailed Description/Instructions:** Students will be given a note card with a polynomial expression on it, and they will be expected to analyze the degree, number of terms, and leading coefficient. In the first round, students will organize themselves into groups by degree, answering questions involving adding and subtracting polynomials. In the second round, students will organize themselves by number of terms and will multiply polynomials. In the third round, students will find groups based on the leading coefficient and will simplify expressions in quadratic form and using division.

Bloom's Levels: Analyze Webb's DOK: 2

## Topic 2: Factoring

**Engaging Experience 1** 

Title: Factoring Using Algebra Tiles

Suggested Length of Time: 10 minutes

**Standards Addressed** 

Priority:

• Alg1.SSE.A.2: Analyze the structure of polynomials to create equivalent expressions or equations.

Supporting:

- Alg2.FM.A.1: Create functions and use them to solve applications of quadratic and exponential function model problems.
- ISTE-COMPUTATIONAL THINKER.5.C break problems into component parts, extract key information, and develop descriptive models to understand complex systems or facilitate problem-solving.

**Detailed Description/Instructions:** Students will be given the task of creating a rectangle given algebra tiles or creating their own tiles using Smart Note book. For example, the equation x2+8x+15would be represented by one blue square, 8 green "rows of *x*" and 15 green single tiles. Using those tiles, students will attempt to create a rectangle. In this activity, students will see the relationship between factoring and FOILing and start to learn what it means to both be a perfect square trinomial and complete the square.

Bloom's Levels: Analyze Webb's DOK: 2

## **Engaging Scenario**

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will pair up to design a swimming pool complex using specific guidelines and a budget. Students will be required to multiply polynomials, factor, and solve quadratics by factoring in a real-world scenario.

# Summary of Engaging Learning Experiences for Topics

Торіс	Engaging Experience Title	Description	Suggested Length of Time
Simplify Polynomials	Polynomial Operation Mix- And-Match Activity	Students will be given a note card with a polynomial expression on it, and they will be expected to analyze the degree, number of terms, and leading coefficient. In the first round, students will organize themselves into groups by degree, answering questions involving adding and subtracting polynomials. In the second round, students will organize themselves by number of terms and will multiply polynomials. In the third round, students will find groups based on the leading coefficient and will simplify expressions in quadratic form and using division.	20 minutes
Factoring	Factoring Using Algebra Tiles	Students will be given the task of creating a rectangle given algebra tiles or creating their own tiles using Smart Note book. For example, the equation x2+8x+15 would be represented by one blue square, 8 green "rows of x" and 15 green single tiles. Using those tiles, students will attempt to create a rectangle. In this activity, students will see the relationship between factoring and FOILing and start to learn what it means to both be a perfect square trinomial and complete the square.	10 minutes

## Unit 5: Rationals

Subject: College Algebra

Grade: 10, 11, 12

Name of Unit: Rationals

Length of Unit: 6 class periods

**Overview of Unit:** In this unit students will simplify, add, subtract, multiply, and divide rational functions. Students will also solve rational equations.

### **Priority Standards for unit:**

- Alg2.REI.A.2: Solve rational equations where numerators and denominators are polynomials and where extraneous solutions may result.
- Alg2.APR.A.4: Add, subtract, multiply and divide rational expressions.

### Supporting Standards for unit:

- Alg2.APR.A.2: Understand the Remainder Theorem and use it to solve problems.
- Alg2.APR.A.3: Find the least common multiple of two or more polynomials.
- ISTE-EMPOWERED LEARNER1.C use technology to seek feedback that informs and improves their practice and to demonstrate their learning in a variety of ways.
- ISTE-GLOBAL COLLABORATOR.7.C contribute constructively to project teams, assuming various roles and responsibilities to work effectively toward a common goal.

Unwrapped Concepts (Students need to know)	Unwrapped Skills (Students need to be able to do)	Bloom's Taxonomy Levels	Webb's DOK
rational equations where			
numerators and denominators are			
polynomials and where extraneous			
solutions may result	Solve	Analyze	3
rational expressions	Add, subtract, multiply and divide	Apply	1

### **Essential Questions**:

- 1. How do you simplify rational expressions?
- 2. Why is the domain affected by rational functions?
- 3. Why are the domain restrictions in rational equations important?
- 4. How do you solve rational equations and use them to model real-world applications?

### **Enduring Understanding/Big Ideas:**

1. Add, subtract, multiply, and divide using the cancellation property and common denominators.

- 2. A domain restriction occurs when there are values for *x* that make the function undefined. Rational functions have undefined values because they make the denominator equal to zero.
- 3. Domain restrictions create potentially extraneous solutions.
- 4. Find the domain restrictions, multiply by the LCD, and solve for the variable. Check for extraneous solutions by analyzing the domain restrictions.

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Academic Cross-Curricular Words	Content/Domain Specific
Isolate variable	Domain restrictions
	Complex fractions
	Least Common Denominator
	Least Common Divisor
	Extraneous
	Cancellation Property

**Resources for Vocabulary Development:** textbook

## Topic 1: Simplify Rational Expressions

### **Engaging Experience 1**

Title: Quadrant Partners Activity Suggested Length of Time: 20 minutes Standards Addressed

Priority:

• Alg2.APR.A.4: Add, subtract, multiply and divide rational expressions.

Supporting:

- Alg2.APR.A.3: Find the least common multiple of two or more polynomials.
- ISTE-EMPOWERED LEARNER1.C use technology to seek feedback that informs and improves their practice and to demonstrate their learning in a variety of ways.

**Detailed Description/Instructions:** Students will find four partners around the room (a different individual for each of their quadrants). In their partnership, students will work together to solve a problem put the solution in an online document such as the collaboration space on One Note, and the teacher will ensure each group has the correct solution, addressing any misconceptions as they arise. This process will be repeated for each of the four quadrant partners. **Bloom's Levels:** Apply

Webb's DOK: 1

### **Engaging Experience 2**

**Title:** RallyCoach Activity **Suggested Length of Time:** 10 minutes **Standards Addressed** 

Priority:

• Alg2.APR.A.4: Add, subtract, multiply and divide rational expressions. *Supporting*:

• Alg2.APR.A.3: Find the least common multiple of two or more polynomials. **Detailed Description/Instructions:** Students will find a partner and complete four problems. Two of the problems will be completed by Partner A, with Partner B checking for accuracy. Similarly, two problems will be completed by Partner B, with Partner A checking for accuracy. This accountability piece will hold each partner accountable, even if he or she is not currently working a problem.

Bloom's Levels: Apply Webb's DOK: 1

## Topic 2: Solve and Apply



**Engaging Experience 1** 

Title: Round Table Activity

**Suggested Length of Time:** 45 minutes

**Standards Addressed** 

Priority:

• Alg2.REI.A.2: Solve rational equations where numerators and denominators are polynomials and where extraneous solutions may result.

Supporting:

- Alg2.APR.A.2: Understand the Remainder Theorem and use it to solve Problems.
- ISTE-GLOBAL COLLABORATOR.7.C contribute constructively to project teams, assuming various roles and responsibilities to work effectively toward a common goal.

**Detailed Description/Instructions:** Students will be placed in groups of four by the teacher and presented with a problem involving modeling a real-world scenario using rational equations. Each student will have one of four unique roles. He or she will either define the variables/ provide a diagram, set up the equation, solve the equation, or effectively communicate the solution to the problem. This process will be repeated four times so that each team member can experience each role in the group, with the teacher checking the solution before providing the next problem to each group.

Bloom's Levels: Analyze Webb's DOK: 3

### **Engaging Scenario**

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will be given the choice of a task (work out simple math problems, shoot baskets, stack blocks, etc.). Students will then individually complete their task and record the time it took for them to complete given task. Students will then pair up and, based on their individual times, calculate how long it would take them to complete the task together. Students will then complete the task together, record the time it took for them to complete the task together, record the time it took for them to complete the task together.

# Summary of Engaging Learning Experiences for Topics

Торіс	Engaging Experience Title	Description	Suggested Length of Time
Simplify Rational Expressions	Quadrant Partners Activity	Students will find four partners around the room (a different individual for each of their quadrants). In their partnership, students will work together to solve a problem put the solution in an online document such as the collaboration space on One Note, and the teacher will ensure each group has the correct solution, addressing any misconceptions as they arise. This process will be repeated for each of the four quadrant partners.	20 minutes
Simplify Rational Expressions	RallyCoach Activity	Students will find a partner and complete four problems. Two of the problems will be completed by Partner A, with Partner B checking for accuracy. Similarly, two problems will be completed by Partner B, with Partner A checking for accuracy. This accountability piece will hold each partner accountable, even if he or she is not currently working a problem.	10 minutes
Solve and Apply	Round Table Activity	Students will be placed in groups of four by the teacher and presented with a problem involving modeling a real-world scenario using rational equations. Each student will have one of four unique roles. He or she will either define the variables/ provide a diagram, set up the equation, solve the equation, or effectively communicate the solution to the problem. This process will be repeated four times so that each team member can experience each role in the group, with the teacher checking the solution before providing the next problem to each group.	45 minutes

## Unit 6: Radicals

Subject: College Algebra

**Grade:** 10, 11, 12 **Name of Unit:** Radicals **Length of Unit:** 6 class periods

**Overview of Unit:** In this unit students will work with radicals and rational exponents. They will also be introduced to complex numbers and operations with complex numbers.

### **Priority Standards for unit:**

- Alg2.NQ.A.1: Extend the system of powers and roots to include rational exponents.
- Alg2.NQ.A.2: Create and recognize equivalent expressions involving radical and exponential forms of expressions.
- Alg2.NQ.A.3: Add, subtract, multiply and divide radical expressions.
- Alg2.NQ.A.4: Solve equations involving rational exponents and/or radicals and identify situations where extraneous solutions may result.

### Supporting Standards for unit:

- Alg2.NQ.B.1: Represent complex numbers.
- Alg2.NQ.B.2: Add, subtract, multiply and divide complex numbers.
- Alg2.IF.A.2: Translate between equivalent forms of functions.
- NMP.FF.2 Use multiple representations of functions to interpret and describe how two quantities change together.
- ISTE-KNOWLEDGE COLLECTOR.3.B evaluate the accuracy, perspective, credibility and relevance of information, media, data or other resources.
- ISTE-EMPOWERED LEARNER1.D understand the fundamental concepts of technology operations, demonstrate the ability to choose, use and troubleshoot current technologies and are able to transfer their knowledge to explore emerging technologies.

		Bloom's	
Unwrapped Concepts	<b>Unwrapped Skills</b>	Taxonomy	Webb's
(Students need to know)	(Students need to be able to do)	Levels	DOK
the system of powers and roots to			
include rational exponents.	Extend	Understand	4
equivalent expressions involving			
radical and exponential forms of			
expressions.	Create	Create	1
equivalent expressions involving			
radical and exponential forms of			
expressions.	Recognize	Create	2

	Add, subtract, multiply, and		
radical expressions.	divide	Apply	1
equations involving rational			
exponents and/or radicals	Solve	Apply	2
situations where extraneous			
solutions may result	Identify	Analyze	3

### Essential Questions:

- 1. How do you simplify radicals?
- 2. How do you combine radicals using the basic operations?
- 3. How do you rewrite radicals using rational exponents?
- 4. How do you solve equations involving radicals?
- 5. What is a complex number and how do you simplify complex numbers?

### **Enduring Understanding/Big Ideas:**

- 1. You can simplify radicals by writing a radical in simplest form including the proper use of absolute values.
- 2. Adding, subtracting, multiplying and dividing radicals, including rationalizing the denominator.
- 3. Rewriting a radical using rational exponents, and rewriting a number with a rational exponent as a radical.
- 4. Isolate the variable by applying the order of operations backwards and checking for extraneous solutions.
- 5. Define a complex number and do the basic mathematical operations with complex numbers.

### **Unit Vocabulary:**

Academic Cross-Curricular Words	<b>Content/Domain Specific</b>
	Radical
	Rational Exponents
	Rationalize
	Extraneous
	Complex Numbers
	Conjugate
	Radicand
	Index
	Root
	Square Root

**Resources for Vocabulary Development:** Textbook

# Topic 1: Simplifying Radicals

#### **Engaging Experience 1**

Title: Guided Practice

Suggested Length of Time: 15 minutes

**Standards Addressed** 

Priority:

- ALG2.NQ.A.1: Extend the system of powers and roots to include rational exponents.
- ALG2.NQ.A.2: Create and recognize equivalent expressions involving radical and exponential forms of expressions.

Supporting:

• Alg2.IF.A.2: Translate between equivalent forms of functions.

**Detailed Description/Instructions:** Students will review the process of simplifying radicals by performing various examples of simplifying radicals on a personal whiteboard (or using their laptop as a whiteboard).

**Bloom's Levels:** Understand, Create, Create **Webb's DOK:** 4, 1, 2

# **Topic 2: Solving Radical Equations**

**Engaging Experience 1** 

Title: Who is Right?

Suggested Length of Time: 10 minutes

Standards Addressed

Priority:

• Alg2.NQ.A.4: Solve equations involving rational exponents and/or radicals and identify situations where extraneous solutions may result.

Supporting:

- NMP.FF.2 Use multiple representations of functions to interpret and describe how two quantities change together.
- ISTE-KNOWLEDGE COLLECTOR.3.B evaluate the accuracy, perspective, credibility and relevance of information, media, data or other resources.

**Detailed Description/Instructions:** Give students a problem (sqrt of x = -3). Show two students solutions - Carlos says the answer is 9 because you are supposed to square both sides, Andrea says there is no solution, have students discuss who is right and why - be sure to emphasize the importance of checking for extraneous solutions. Then move on to a more difficult example (6 = x + sqrt(x)). Have students solve them and compare their solutions with a partner. **Bloom's Levels:** Apply **Webb's DOK:** 1

# Topic 3: Complex Numbers

**Engaging Experience 1** 

Title: Who is Right?

Suggested Length of Time: 10-15 minutes

**Standards Addressed** 

Priority:

• Alg2.NQ.A.3: Add, subtract, multiply and divide radical expressions

Supporting:

- Alg2.NQ.B.1: Represent complex numbers.
- Alg2.NQ.B.2: Add, subtract, multiply and divide complex numbers.
- ISTE-KNOWLEDGE COLLECTOR.3.B evaluate the accuracy, perspective, credibility and relevance of information, media, data or other resources.

**Detailed Description/Instructions:** After reviewing the definition of a complex number present students with the following problem:  $sqrt(-4) \ge 4$ . Discuss as a class or in small groups whether this is correct or incorrect and why. Then go on to explain, if necessary, that if  $sqrt(3) \ge 3$ , shouldn't the  $sqrt(-4) \ge -4$ ? Using complex numbers show why this works. **Bloom's Levels:** Apply **Webb's DOK:** 1

## **Engaging Scenario**

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will access the graphing calculator features on Desmos. They will input various nth roots of the nth powers to notice patterns. Through discussion and collaboration, we will determine when the graphs are strictly located above the *x*-axis. Then, we will reverse the process to see when extraneous solutions are needed.

# Summary of Engaging Learning Experiences for Topics

Торіс	Engaging Experience Title	Description	Suggested Length of Time
Simplifying Radicals	Guided Practice	Students will review the process of simplifying radicals by performing various examples of simplifying radicals on a personal whiteboard (or using their laptop as a whiteboard).	15 minutes
Solving Radical Equations	Who is Right?	Give students a problem (sqrt of $x = -3$ ). Show two students solutions - Carlos says the answer is 9 because you are supposed to square both sides, Andrea says there is no solution, have students discuss who is right and why - be sure to emphasize the importance of checking for extraneous solutions. Then move on to a more difficult example ( $6 = x + \text{sqrt}(x)$ ). Have students solve them and compare their solutions with a partner.	10 minutes
Complex Numbers	Who is Right?	After reviewing the definition of a complex number present students with the following problem: sqrt(-4) x sqrt(-4) = 4. Discuss as a class or in small groups whether this is correct or incorrect and why. Then go on to explain, if necessary, that if sqrt(3) x sqrt(3) = 3, shouldn't the sqrt(-4) x sqrt(-4) = -4? Using complex numbers show why this works.	10-15 minutes

# Unit 7: Quadratics

Subject: College Algebra

**Grade:** 10, 11, 12 **Name of Unit:** Quadratics **Length of Unit:** 6 class periods

**Overview of Unit**: In this unit students will solve quadratic (and quadratic type) functions, analyze the graph of a quadratic function and solve quadratic inequalities.

### **Priority Standards for unit:**

• Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

#### **Supporting Standards for unit:**

- Alg2.IF.A.2: Translate between equivalent forms of functions.
- Alg2.FM.A.1: Create functions and use them to solve applications of quadratic and exponential function model problems.
- NMP.FF.2 Use multiple representations of functions to interpret and describe how two quantities change together.
- ISTE-KNOWLEDGE COLLECTOR.3.D build knowledge by actively exploring realworld issues and problems, developing ideas and theories and pursuing answers and solutions.
- ISTE-COMPUTATIONAL THINKER.5.B collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.
- Alg1.SSE.A.3: Choose and produce equivalent forms of a quadratic expression or equations to reveal and explain properties.
  - Find the zeros of a quadratic function by rewriting it in factored form.
  - Find the maximum or minimum value of a quadratic function by completing the square.

Unwrapped Concepts (Students need to know)	Unwrapped Skills (Students need to be able to do)	Bloom's Taxonomy Levels	Webb's DOK
key characteristics of functions represented			
graphically, with tables and with algebraic			
symbolism to solve problems.	Identify	Understand	2
key characteristics of functions represented			
graphically, with tables and with algebraic			
symbolism to solve problems.	Interpret	Analyze	3

### **Essential Questions:**

- 1. How do you solve a quadratic function?
- 2. How do you use the graph of a quadratic function to find the maximum/minimum values?
- 3. How do you solve a quadratic inequality?
- 4. How do you write a quadratic equation from given roots?
- 5. How do you solve other polynomials in quadratic form?

#### **Enduring Understanding/Big Ideas:**

- 1. Solve a quadratic function using a variety of techniques including factoring, the quadratic formula and completing the square.
- 2. Use the vertex to graph a quadratic function and find the maximum or minimum value of that function.
- 3. Solve a quadratic inequality by graphing and using a table and then communicating the solution using interval notation.
- 4. Work backwards to write a quadratic equation from given roots.
- 5. Solving quadratic type equations using substitution.

#### **Unit Vocabulary:**

Academic Cross-Curricular Words	<b>Content/Domain Specific</b>	
Roots	Quadratic	
Optimization (Maximum/Minimum)	Discriminant	
Intercepts	Quadratic inequalities	
	Factoring	
	Parabola	
	Vertex	

**Resources for Vocabulary Development:** Textbook

# **Topic 1: Polynomial Functions**

### **Engaging Experience 1**

**Title:** Generate the quadratic formula by completing the square **Suggested Length of Time:** 10 minutes **Standards Addressed** 

Priority:

• Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

Supporting:

- Alg2.IF.A.2: Translate between equivalent forms of functions
- Alg2.FM.A.1: Create functions and use them to solve applications of quadratic and exponential function model problems.
- Alg1.SSE.A.3: Choose and produce equivalent forms of a quadratic expression or equations to reveal and explain properties.
   Find the zeros of a quadratic function by rewriting it in factored form.
   Find the maximum or minimum value of a quadratic function by completing the square.
- ISTE-KNOWLEDGE COLLECTOR.3.D build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.

**Detailed Description/Instructions:** Students can complete this on a white board: Have the students write out a quadratic equation in standard form (with values for a, b, and c where a does not equal 1). Have them solve this equation by completing the square. Now have them write out  $ax^2 + bx + c = 0$  and solve this by completing the square. If they do it correctly, they should generate the quadratic formula.

**Bloom's Levels:** Understand, Analyze **Webb's DOK:** 2, 3

# Topic 2: Solving and Applying

**Engaging Experience 1** 

**Title:** Making a connection **Suggested Length of Time:** 20 minutes **Standards Addressed** 

Priority:

• Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

Supporting:

- NMP.FF.2 Use multiple representations of functions to interpret and describe how two quantities change together.
- ISTE-COMPUTATIONAL THINKER.5.B collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.

**Detailed Description/Instructions:** Give the students a quadratic inequality and have them graph it on their graphing calculator (or use Desmos), demonstrate the solution using interval notation. Give them a few more examples with a variety of solution types and a variety of types of quadratic functions (some in factored form). Discuss what conclusions can be made about the solutions (they should notice that the critical points come from the roots of the quadratic equations). Now demonstrate the test-point method as another way of solving a quadratic inequality without graphing on a coordinate plane.

**Bloom's Levels:** Understand, Analyze **Webb's DOK:** 2, 3

## **Engaging Scenario**

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) "Match My Curve!" activity on Desmos. In this activity, students develop their understanding of standard, factored, and vertex forms of quadratic functions by matching parabolas (as closely as they can) to curves in images from the real world.

# Summary of Engaging Learning Experiences for Topics

Торіс	Engaging Experience Title	Description	Suggested Length of Time
Polynomial Functions	Generate the quadratic formula by completing the square	Students can complete this on a white board: Have the students write out a quadratic equation in standard form (with values for a, b, and c where a does not equal 1). Have them solve this equation by completing the square. Now have them write out $ax^2 + bx + c = 0$ and solve this by completing the square. If they do it correctly, they should generate the quadratic formula.	10 minutes
Solving and Applying	Making a connection	Give the students a quadratic inequality and have them graph it on their graphing calculator (or use Desmos), demonstrate the solution using interval notation. Give them a few more examples with a variety of solution types and a variety of types of quadratic functions (some in factored form). Discuss what conclusions can be made about the solutions (they should notice that the critical points come from the roots of the quadratic equations). Now demonstrate the test-point method as another way of solving a quadratic inequality without graphing on a coordinate plane.	20 minutes

# Unit 8: Functions

Subject: College Algebra

Grade: 10, 11, 12

Name of Unit: Functions

Length of Unit: 9 class periods

**Overview of Unit:** Students will graph a variety of functions use domain, range, tables, and transformations. Students will use function notation to combine functions, find average rate of change, and determine if a function has symmetry. Students will find inverses to a function algebraically, graphically and numerically.

## **Priority Standards for unit:**

- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.
- Alg2.BF.A.1: Create new functions by applying the four arithmetic operations and composition of functions (modifying the domain and range as necessary).
- Alg2.BF.A.2: Derive inverses of functions, and compose the inverse with the original function to show that the functions are inverses.

## Supporting Standards for unit:

- Alg2.BF.A.3: Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for linear, quadratic, cubic, square and cube root, absolute value, exponential and logarithmic functions.
- NMP.FF.2 Use multiple representations of functions to interpret and describe how two quantities change together.
- ISTE-EMPOWERED LEARNER1.D understand the fundamental concepts of technology operations, demonstrate the ability to choose, use and troubleshoot current technologies and are able to transfer their knowledge to explore emerging technologies
- ISTE-KNOWLEDGE COLLECTOR.3.D build knowledge by actively exploring realworld issues and problems, developing ideas and theories and pursuing answers and solutions.

Unwrapped Concepts (Students need to know)	Unwrapped Skills (Students need to be able to do)	Bloom's Taxonomy Levels	Webb's DOK
key characteristics of functions represented			
graphically, with tables and with algebraic			
symbolism to solve problems.	Identify	Understand	2

Interpret	Analyze	3
Create	Create	2
Apply	Apply	1
Derive	Apply	2
Compose	Apply	3
	Create Apply Derive	Create Create Apply Apply Derive Apply

### **Essential Questions:**

- 1. How can you represent a function in a variety of ways?
- 2. How can you find the domain and range of a function and use that to graph a given function?
- 3. How can you analyze the graph of a function?
- 4. How do you combine functions?
- 5. How do you find the inverse of a function?

#### **Enduring Understanding/Big Ideas:**

- 1. Represent a function verbally, algebraically, graphically and numerically, using function notation when appropriate.
- 2. Determine the domain and range of a function, use the information to graph a variety of functions including piecewise functions.
- 3. Using the graph of a function to apply transformations, find optimal values and determine the average rate of change.
- 4. Use the basic operations to combine functions to obtain new functions and finding the composition of functions.
- 5. Finding the inverse of 1-1 functions and proving that functions are inverses of each other.

## **Unit Vocabulary:**

Academic Cross-Curricular Words	<b>Content/Domain Specific</b>	
Function	Domain	
Independent Variable	Range	
Dependent Variable	Piecewise Function	
Net Change	Even and Odd Functions	
Maxima	Composition	
Minima	One to One	
Average Rate of Change	Horizontal Line Test	
Transformations		
Inverse		

**Resources for Vocabulary Development:** Textbook

# Topic 1: Functions and Graphs

**Engaging Experience 1** 

Title: Just Move it - Desmos Suggested Length of Time: 45 minutes Standards Addressed

Priority:

• Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

Supporting:

- Alg2.BF.A.3: Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for linear, quadratic, cubic, square and cube root, absolute value, exponential and logarithmic functions.
- NMP.FF.2 Use multiple representations of functions to interpret and describe how two quantities change together.
- ISTE-EMPOWERED LEARNER1.D understand the fundamental concepts of technology operations, demonstrate the ability to choose, use and troubleshoot current technologies and are able to transfer their knowledge to explore emerging technologies.

**Detailed Description/Instructions:** Students will complete the Move It - Transformations activity on Desmos.

**Bloom's Levels:** Understand and Analyze **Webb's DOK**: 2, 3

# Topic 2: Analyzing Functions

**Engaging Experience 1** 

Title: Cell Phone Activity

Suggested Length of Time: 30 minutes

**Standards Addressed** 

Priority:

• Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

Supporting:

- NMP.FF.2 Use multiple representations of functions to interpret and describe how two quantities change together.
- ISTE-KNOWLEDGE COLLECTOR.3.D build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.

**Detailed Description/Instructions:** After being introduced to the idea of a piecewise function students will be given a cell phone scenario with 2-3 different plans (for example plan A: \$30 monthly fee, 450 anytime minutes, \$.45 for each additional minute, plan B: \$45 monthly fee, 900 anytime minutes, \$.30 for each additional minute). Students will be given a task to create a piecewise function to represent each scenario, then answer questions about which is the best plan if you use 400 minutes each month, 475, 600, etc., verifying this both graphically and algebraically.

**Bloom's Levels:** Understand, Analyze **Webb's DOK:** 2, 3

# Topic 3: Combining Functions

**Engaging Experience 1** 

**Title:** Chain Reaction **Suggested Length of Time:** 15 minutes **Standards Addressed** 

Priority:

• Alg2.BF.A.1: Create new functions by applying the four arithmetic operations and composition of functions (modifying the domain and range as necessary).

Supporting:

• NMP.FF.2 Use multiple representations of functions to interpret and describe how two quantities change together.

**Detailed Description/Instructions:** Students will be divided into groups of 4 or 5 and labeled with a number 1-5 (for groups of 4 one person will go twice). The teacher will prepare 5 functions (using a variety of ones discussed - quadratic, linear, square root, etc.) and print these same 5 functions on note cards for each of the groups in the class. Each group will receive the set of 5 functions and distribute them to their group members in random order. The teacher will then call out a number, for example -1. The 1 student in each group will evaluate his/her function at that value, the 2 students will then evaluate his/her function at the answer to student 1's function. This will continue on until all 5 functions have been evaluated. Answers will be checked, and students will then have an opportunity to go back and fix their chain if they were wrong. This is a good lead into discussing the composition of functions.

Bloom's Levels: Apply Webb's DOK: 2

# **Topic 4: Inverse Functions**

#### **Engaging Experience 1**

Title: Inverse function partner share Suggested Length of Time: 30 minutes Standards Addressed

Priority:

• Alg2.BF.A.2: Derive inverses of functions, and compose the inverse with the original function to show that the functions are inverses.

Supporting:

• NMP.FF.2 Use multiple representations of functions to interpret and describe how two quantities change together

**Detailed Description/Instructions:** The teacher will begin by writing a function on the board (for example  $3x^2 + 1$ ) and as a class they should generate the sequence of steps needed to evaluate the function (1 - square the input, 2 - multiply by 3, 3- add 1). From here the class will divide into teams of 2. Each team of two will be given two different functions that are not inverses of each other, however another team should have functions that are inverses of another team's functions. Students should write out the sequence of steps for each of their functions and provide 3 ordered pairs which satisfy their functions. Once each team is done, they should seek out a team that mirrors their steps. Once they have found their other half, they should discuss what they notice about their functions (the steps are in the opposite order - steps are "undone", they should notice their ordered pairs are also reversed). This should lead to a class discussion of what an inverse is, the process of finding an inverse, and the domain-range relationship between functions and their inverses.

**Bloom's Levels:** Apply, Apply Webb's DOK: 2, 3

## **Engaging Scenario**

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Use Function Carnival from Desmos Teacher Activities to have students put different parent functions together in a piecewise function. Students will also need to use the concepts of increasing and decreasing intervals to identify where to place maximum and minimum points.

# Summary of Engaging Learning Experiences for Topics

Торіс	Engaging Experience Title	Description	Suggested Length of Time
Functions and Graphs	Just Move it – Desmos	Students will complete the Move It - Transformations activity on Desmos.	45 minutes
Analyzing Functions	Cell Phone Activity	After being introduced to the idea of a piecewise function students will be given a cell phone scenario with 2 - 3 different plans (for example plan A: \$30 monthly fee, 450 anytime minutes, \$.45 for each additional minute, plan B: \$45 monthly fee, 900 anytime minutes, \$.30 for each additional minute). Students will be given a task to create a piecewise function to represent each scenario, then answer questions about which is the best plan if you use 400 minutes each month, 475, 600, etc., verifying this both graphically and algebraically.	30 minutes
Combining Functions	Chain Reaction	Students will be divided into groups of 4 or 5 and labeled with a number 1-5 (for groups of 4 one person will go twice). The teacher will prepare 5 functions (using a variety of ones discussed - quadratic, linear, square root, etc.) and print these same 5 functions on note cards for each of the groups in the class. Each group will receive the set of 5 functions and distribute them to their group members in random order. The teacher will then call out a number, for example -1. The 1 student in each group	15 minutes

# Unit 9: Polynomials and Rationals

Subject: College Algebra

Grade: 10, 11, 12

Name of Unit: Polynomials and Rationals

Length of Unit: 10 class periods

**Overview of Unit:** Students will graph polynomials and rational functions using domain, end behavior, and intercepts. Students will find all zeros of polynomial including complex zeros and multiplicity of zeros.

## **Priority Standards for unit:**

- Alg2.NQ.B.3: Know and apply the Fundamental Theorem of Algebra.
- Alg2.APR.A.5: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to sketch the function defined by the polynomial.
- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

## Supporting Standards for unit:

- Alg2.APR.A.1: Extend the knowledge of factoring to include factors with complex coefficients.
- Alg2.APR.A.2: Understand the Remainder Theorem and use it to solve problems.
- Alg2.FM.A.1: Create functions and use them to solve applications of quadratic and exponential function model problems.
- NMP.FF.1 Conceptualize quantities and define variables that are present in a given situation.
- NMP.FF.2 Use multiple representations of functions to interpret and describe how two quantities change together.
- NMP.FF.3 Measure, compute, describe, and interpret rates of change of quantities embedded in multiple representations.
- ISTE-COMPUTATIONAL THINKER.5.A formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.
- ISTE-KNOWLEDGE COLLECTOR.3.A plan and employ effective research strategies to locate information and other resources for their intellectual or creative pursuits.

Unwrapped Concepts (Students need to know)	Unwrapped Skills (Students need to be able to do)		Webb's DOK
the Fundamental Theorem of Algebra	Know	Remember	1
the Fundamental Theorem of Algebra	Apply	Apply	3

zeros of polynomials when suitable			
factorizations are available	Identify	Understand	1
the zeros to sketch the function defined by the			
polynomial.	Use	Understand	2
key characteristics of functions represented			
graphically, with tables and with algebraic			
symbolism to solve problems.	Identify	Understand	2
key characteristics of functions represented			
graphically, with tables and with algebraic			
symbolism to solve problems.	Interpret	Analyze	3

#### **Essential Questions:**

- 1. How can the number of solutions to a polynomial be determined?
- 2. How can the end behavior of a polynomial be determined?
- 3. How do domain issues affect the graph of a rational function?
- 4. How can the end behavior of a rational function be determined?

#### **Enduring Understanding/Big Ideas:**

- 1. The degree of the polynomial determines the number of the solutions.
- 2. The degree and Leading Coefficient determine the end behavior.
- 3. Values not in the domain are either a hole or a vertical asymptote.
- 4. The end behavior will be determined by dividing quotient.

#### **Unit Vocabulary:**

Academic Cross-Curricular Words	<b>Content/Domain Specific</b>	
Maximum	End behavior	
Minimum	Zero	
Domain	Polynomial	
Range	Rational	
Quotient	Synthetic Division	
Remainder	Degree	
Joint Variation	Leading Coefficient	
Inverse Variation	Asymptote	
Direct Variation	Quadratic	
Proportional	Complex Number	
Constant	Roots	
	Multiplicity	
	Intercepts	
	Bounds	
	Conjugate	

#### Resources for Vocabulary Development: textbook

# **Topic 1: Polynomial Functions**

#### **Engaging Experience 1**

Title: Real World Extrema Problems

#### Suggested Length of Time: 1/2 of a class period

### **Standards Addressed**

Priority:

- Alg2.APR.A.5: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to sketch the function defined by the polynomial.
- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

Supporting:

- Alg2.FM.A.1: Create functions and use them to solve applications of quadratic and exponential function model problems.
- NMP.FF.1 Conceptualize quantities and define variables that are present in a given situation.

**Detailed Description/Instructions:** Students will be given different scenarios that require them to write a quadratic function to represent the problem. They will then use the equation to determine the optimal solution (max profit, min fencing, etc...)

Bloom's Levels: Identify, Use, Analyze

Webb's DOK: 1, 2, 3 Rubric: To be created

#### **Engaging Experience 2**

**Title:** Analyze the graphs of polynomials **Suggested Length of Time:** 1 class period **Standards Addressed** 

Priority:

- Alg2.APR.A.5: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to sketch the function defined by the polynomial.
- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

Supporting:

• ISTE-COMPUTATIONAL THINKER.5.A - formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.

**Detailed Description/Instructions:** Students will use end behavior and multiplicity of zeros to graph several polynomials. The students then will use technology (graphing calculator or Desmos) to verify their answers.

**Bloom's Levels:** Identify, Use, Analyze **Webb's DOK:** 1, 2, 3

## **Engaging Experience 3**

**Title**: Finding Zeros of Polynomials **Suggested Length of Time:** 2 class periods **Standards Addressed** 

Priority:

- Alg2.IF.A.1:
- Alg2.NQ.B.3: Know and apply the Fundamental Theorem of Algebra.
- Alg2.APR.A.5: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to sketch the function defined by the polynomial.

Supporting:

- Alg2.APR.A.1: Extend the knowledge of factoring to include factors with complex coefficients.
- Alg2.APR.A.2: Understand the Remainder Theorem and use it to solve problems.
- ISTE-KNOWLEDGE COLLECTOR.3.A plan and employ effective research strategies to locate information and other resources for their intellectual or creative pursuits.

**Detailed Description/Instructions:** Students will be given a variety of polynomial functions and be asked to use technology and theorems (possible rational zeros, Descartes' Rule of signs, and bounds) to determine potential zeros of the function. They will then use synthetic or polynomial division to reduce the function and identify zeros. Some zeros will be complex.

Bloom's Levels: Analyze

Webb's DOK: 3

# Topic 2: Rational Functions

**Engaging Experience 1** 

Title: Analyze Rational Functions

Suggested Length of Time: 3/4 of a class period

## **Standards Addressed**

Priority:

• Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

Supporting:

• ISTE-COMPUTATIONAL THINKER.5.A - formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.

**Detailed Description/Instructions:** Students will be tasked with taking a variety of rational functions and analyzing them to find domain, range, holes, asymptotes, end behavior, intercepts, and extrema. They will then create graphs of these functions both by hand and with the assistance of technology.

**Bloom's Levels:** Analyze **Webb's DOK:** 3

## **Engaging Experience 2**

**Title:** Variation **Suggested Length of Time:** 1/2 of a class period **Standards Addressed** 

Priority:

• Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

Supporting:

- NMP.FF.2 Use multiple representations of functions to interpret and describe how two quantities change together.
- NMP.FF.3 Measure, compute, describe, and interpret rates of change of quantities embedded in multiple representations.

**Detailed Description/Instructions:** Students will work on getting a better understanding of the relationship between variable. Students will be given a scenario, and will find an equation and determine future data points using that equation. Potential scenarios could include: applying gas laws, or the gravitational force between two masses, or scenarios involving movement and examining acceleration or deceleration.

Bloom's Levels: Analyze Webb's DOK: 3

## **Engaging Scenario**

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will be broken into groups and given sheets of paper of various sizes. They will construct an open topped box of maximum volume. Their product will be a physical box, the equation modeling their boxes volume, a screenshot of technology showing values of the maximum, and a justification of their findings.

The students then will find the two values they could have cut out to make a box of half the volume. If their solution can be found without technology, they must show the work by hand.

# Summary of Engaging Learning Experiences for Topics

Торіс	Engaging Experience Title	Description	Suggested Length of Time
Polynomial Functions	Real World Extrema Problems	Students will be given different scenarios that require them to write a quadratic function to represent the problem. They will then use the equation to determine the optimal solution (max profit, min fencing, etc)	<sup>1</sup> ⁄2 of a class period
Polynomial Functions	Analyze the graphs of polynomials	Students will use end behavior and multiplicity of zeros to graph several polynomials. The students then will use technology (graphing calculator or Desmos) to verify their answers.	1 class period
Polynomial Functions	Finding Zeros of Polynomials	Students will be given a variety of polynomial functions and be asked to use technology and theorems (possible rational zeros, Descartes' Rule of signs, and bounds) to determine potential zeros of the function. They will then use synthetic or polynomial division to reduce the function and identify zeros. Some zeros will be complex.	2 class periods
Rational Functions	Analyze Rational Functions	Students will be tasked with taking a variety of rational functions and analyzing them to find domain, range, holes, asymptotes, end behavior, intercepts, and extrema. They will then create graphs of these functions both by hand and with the assistance of technology.	<sup>3</sup> ⁄4 of a class period
Rational Functions	Variation	Students will work on getting a better understanding of the relationship between variable. Students will be given a scenario and will find an equation and determine future data points using that equation. Potential scenarios could include applying gas laws, or the gravitational force between two masses, or scenarios involving movement and examining acceleration or deceleration.	<sup>1</sup> /2 of a class period

# Unit 10: Exponentials and Logarithms

Subject: College Algebra

**Grade:** 10, 11, 12

Name of Unit: Exponentials and Logarithms

Length of Unit: 9 class periods

**Overview of Unit:** Students will graph exponential and logarithmic functions using characteristics of each function. Students will solve exponential and logarithmic equations. Students will apply exponential and logarithmic functions to solve real world applications.

## **Priority Standards for unit:**

- Alg2.SSE.A.1: Develop the definition of logarithms based on properties of exponents.
- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

## **Supporting Standards for unit:**

- Alg2.SSE.A.2: Use the inverse relationship between exponents and logarithms to solve exponential and logarithmic equations.
- Alg2.SSE.A.3: Use properties of logarithms to solve equations or find equivalent expressions.
- Alg2.SSE.A.4: Understand why logarithmic scales are used, and use them to solve problems.
- Alg2.IF.A.2: Translate between equivalent forms of functions.
- Alg2.BF.A.3: Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for linear, quadratic, cubic, square and cube root, absolute value, exponential and logarithmic functions.
- Alg2.FM.A.1: Create functions and use them to solve applications of quadratic and exponential function model problems.
- NMP.FF.2 Use multiple representations of functions to interpret and describe how two quantities change together.
- ISTE-COMPUTATIONAL THINKER.5.B collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.
- ISTE-KNOWLEDGE COLLECTOR.3.A plan and employ effective research strategies to locate information and other resources for their intellectual or creative pursuits.

		Bloom's	
Unwrapped Concepts	Unwrapped Skills	Taxonomy	Webb's
(Students need to know)	(Students need to be able to do)	Levels	DOK
the definition of logarithms based			
on properties of exponents	Develop	Understand	4
key characteristics of functions			
represented graphically, with			
tables and with algebraic			
symbolism to solve problems.	Identify	Understand	2
key characteristics of functions			
represented graphically, with			
tables and with algebraic			
symbolism to solve problems.	Interpret	Analyze	3

### **Essential Questions:**

- 1. Why do exponential functions have a horizontal asymptote?
- 2. How are exponential and logarithmic functions related?
- 3. How do we evaluate a logarithmic function? (What is a logarithm?)
- 4. How do we apply exponential functions to the real world?
- 5. How do we apply logarithmic functions to the real world?

#### **Enduring Understanding/Big Ideas:**

- 1. As the exponent gets closer to negative infinity, the value gets closer to 0.
- 2. Exponential and logarithmic functions are inverse functions.
- 3. You rewrite the logarithm as an exponential and find the find the exponent. (It is an exponent.)
- 4. They are used to model exponential growth and compound interest.
- 5. They are used to solve exponential growth and compound interest problems. They are also used to model intensity of sound, earthquakes, and acidity of liquids.

#### **Unit Vocabulary:**

Academic Cross-Curricular Words	Content/Domain Specific
Exponential	Logarithmic
Growth	Asymptote
Decay	Intercepts
Domain	Property
Range	Transformations
Constant	Laws
	Base

Exponent Initial Value
Population Difference

Resources for Vocabulary Development: Textbook

# **Topic 1: Exponential Functions**

**Engaging Experience 1** 

**Title**: Graphing Exponential Functions **Suggested Length of Time:** 1/2 of a class period **Standards Addressed** 

Priority:

• Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

Supporting:

• Alg2.BF.A.3: Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for linear, quadratic, cubic, square and cube root, absolute value, exponential and logarithmic functions.

**Detailed Description/Instructions:** Students will be giving several exponential functions of any base including *e* to graph without the aid of a graphing utility. Students will graph these functions by using transformations and key characteristics common to all exponential functions such as domain, range, the horizontal asymptote, and key points.

Bloom's Levels: Understand

Webb's DOK: 2

# Topic 2: Logarithmic Functions

**Engaging Experience 1** 

**Title:** Develop an Understanding of Logarithms **Suggested Length of Time:** 1/2 of a class period **Standards Addressed** 

Priority:

• Alg2.SSE.A.1: Develop the definition of logarithms based on properties of exponents.

Supporting:

- Alg2.SSE.A.2: Use the inverse relationship between exponents and logarithms to solve exponential and logarithmic equations.
- Alg2.IF.A.2: Translate between equivalent forms of functions.
- NMP.FF.2 Use multiple representations of functions to interpret and describe how two quantities change together.
- Alg2.SSE.A.3: Use properties of logarithms to solve equations or find equivalent expressions.

**Detailed Description/Instructions:** Students will convert logarithmic expressions into exponential functions with an unknown exponent. The students will then use the properties of exponents to solve for the missing exponent.

Bloom's Levels: Understand Webb's DOK: 4

## **Engaging Experience 2**

Title: Graph Logarithms Suggested Length of Time: ½ of a class period Standards Addressed

Priority:

• Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

Supporting:

• Alg2.BF.A.3: Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for linear, quadratic, cubic, square and cube root, absolute value, exponential and logarithmic functions.

**Detailed Description/Instructions:** Students will use the graph of an exponential functions and the properties and symmetry of inverse functions to develop the graphs of logarithms of any base and state key characteristics such as domain, range, vertical asymptote, and key points. The students then will be asked to graph several logarithmic functions using transformations and then compare their graphs to a graphing utility and note the limitations of the graphing utility. **Bloom's Levels:** Understand **Webb's DOK:** 2

## **Engaging Experience 3**

**Title:** Use Properties of Logarithms **Suggested Length of Time:** 1/2 of a class period **Standards Addressed** 

Priority:

• Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

Supporting:

- Alg2.BF.A.3: Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for linear, quadratic, cubic, square and cube root, absolute value, exponential and logarithmic functions.
- ISTE-COMPUTATIONAL THINKER.5.B collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.

**Detailed Description/Instructions:** Students will use the properties of logarithms to expand and condense expressions. To verify their work, students will be given functions and will graph the original functions and the expanded or condensed expression in the same window of a graphing utility to verify the functions are equivalent.

Bloom's Levels: Understand Webb's DOK: 2

# Topic 3: Solve and Apply

### **Engaging Experience 1**

**Title:** Solving Exponential and Logarithmic Equations using a variety of methods **Suggested Length of Time:** 1 class period

### **Standards Addressed**

Priority:

• Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

Supporting:

- Alg2.SSE.A.2: Use the inverse relationship between exponents and logarithms to solve exponential and logarithmic equations.
- Alg2.SSE.A.3: Use properties of logarithms to solve equations or find equivalent expressions.

**Detailed Description/Instructions:** Students will be given exponential and logarithmic functions in a variety of formats. They will be asked to use their knowledge of the inverse relationship of the functions and exponentials and logarithmic properties to solve those equations.

Bloom's Levels: Apply Webb's DOK: 3

## **Engaging Experience 2**

Title: Exponential Growth and Decay Suggested Length of Time: <sup>3</sup>/<sub>4</sub> of a class period Standards Addressed

Priority:

• Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

Supporting:

- Alg2.SSE.A.3: Use properties of logarithms to solve equations or find equivalent expressions.
- Alg2.SSE.A.4: Understand why logarithmic scales are used, and use them to solve problems.
- Alg2.FM.A.1: Create functions and use them to solve applications of quadratic and exponential function model problems.

Detailed Description/Instructions: Students will solve a variety of exponential growth applications. Population growth, radioactive decay, continuously compounded interest, and Newton's Law of Cooling will be included. Bloom's Levels: Interpret Webb's DOK: 3

### **Engaging Scenario**

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will do research to find the retail price of a car 5 years ago and then find the current trade in value. The students will use the data to find the rate of depreciation of the car annually and monthly.

Students then research to find the retail price of a brand-new car. They will then have to find a rate they will be given for a loan to buy that car. Finally, the students will calculate the accumulated amount they will owe if they had not made any payments for 5 years.

# Summary of Engaging Learning Experiences for Topics

Торіс	Engaging Experience Title	Description	Suggested Length of Time
Exponential Functions	Graphing Exponential Functions	Students will be giving several exponential functions of any base including <i>e</i> to graph without the aid of a graphing utility. Students will graph these functions by using transformations and key characteristics common to all exponential functions such as domain, range, the horizontal asymptote, and key points.	<sup>1</sup> ∕₂ of a class period
Logarithmic Functions	Develop an Understanding of Logarithms	Students will convert logarithmic expressions into exponential functions with an unknown exponent. The students will then use the properties of exponents to solve for the missing exponent.	<sup>1</sup> / <sub>2</sub> of a class period
Logarithmic Functions	Graph Logarithms	Students will use the graph of an exponential functions and the properties and symmetry of inverse functions to develop the graphs of logarithms of any base and state key characteristics such as domain, range, vertical asymptote, and key points. The students then will be asked to graph several logarithmic functions using transformations and then compare their graphs to a graphing utility and note the limitations of the graphing utility.	<sup>1</sup> ∕ <sub>2</sub> of a class period
Logarithmic Functions	Use Properties of Logarithms	Students will use the properties of logarithms to expand and condense expressions. To verify their work, students will be given functions and will graph the original functions and the expanded or condensed expression in the same window of a graphing utility to verify the functions are equivalent.	<sup>1</sup> ∕₂ of a class period

Solve and Apply	Solving Exponential and Logarithmic Equations using a variety methods	Students will be given exponential and logarithmic functions in a variety of formats. They will be asked to use their knowledge of the inverse relationship of the functions and exponentials and logarithmic properties to solve those equations.	1 class period
Solve and Apply	Exponential Growth and Decay	Students will solve a variety of exponential growth applications. Population growth, radioactive decay, continuously compounded interest, and Newton's Law of Cooling will be included.	<sup>3</sup> ∕₄ of a class period

## Unit 11: Conics and Systems of Equations

Subject: College Algebra

**Grade:** 10, 11, 12

Name of Unit: Conics and Systems of Equations

Length of Unit: 10 class periods

**Overview of Unit:** Students will graph each conic section using its characteristics. Students will solve systems of equations including nonlinear systems. Students will apply systems of equations and inequalities to solve real world applications including linear programming.

### **Priority Standards for unit:**

- Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.
- Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

### **Supporting Standards for unit:**

- Alg2.IF.A.2: Translate between equivalent forms of functions.
- AL.9-12.F-CS.1 Create graphs of conic sections, including parabolas, hyperbolas, ellipses, circles, and degenerate conics, from second-degree equations.
- AL.9-12.F-CS.1.a Formulate equations of conic sections from their determining characteristics.
- ISTE-COMPUTATIONAL THINKER.5.A formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.
- Write equations of parabolas in vertex form (PC-P.2)
- Write equations of circles in standard form (PC-P.5)
- Write equations of ellipses in standard form (PC-P.9)
- Write equations of hyperbolas in standard form (PC-P.12)
- Convert equations of conic sections from general to standard form (PC-P.13)

Unwrapped Concepts (Students need to know)	Unwrapped Skills (Students need to be able to do)	Bloom's Taxonomy Levels	Webb' s DOK
systems of equations that may			
include non-linear equations and			
inequalities.	Create	Create	3
systems of equations that may			
include non-linear equations and			
inequalities.	Solve	Apply	2

key characteristics of functions			
represented graphically, with			
tables and with algebraic			
symbolism to solve problems.	Identify	Understand	2
key characteristics of functions			
represented graphically, with			
tables and with algebraic			
symbolism to solve problems.	Interpret	Analyze	3

#### **Essential Questions:**

- 1. Why is it important to be able to graph a conic section?
- 2. How solve a real-world problem that has several variables?
- 3. Why is it important to know several methods to solve systems of equations?
- 4. How do I write the solution to a system that has infinite many answers?
- 5. How can systems of inequalities be used to find an optimal solution?

### **Enduring Understanding/Big Ideas:**

- 1. The graphs can be used to solve nonlinear systems.
- 2. Write a system that has as many equations as there are variables.
- 3. Some systems can only be solved graphically or only by substitution. Elimination is the only method that is used when we apply systems to matrices.
- 4. Write the answer using set-builder notation or parametrics.
- 5. They can be used in Linear Programming, to find what needs to be done to find the maximum and minimum values.

### **Unit Vocabulary:**

Academic Cross-Curricular Words	<b>Content/Domain Specific</b>
Circle	Parabola
Nonlinear	Ellipse
	Hyperbola
	Vertex (Vertices)
	Focus (Foci)
	Directrix
	Axis (Major / Minor)
	Focal Diameter
	Asymptote
	System
	Consistent
	Inconsistent

Independent
Dependent
Parametric
Set-Builder
Extrema
Optimization
Feasible Region
Objective Function

**Resources for Vocabulary Development:** Textbook

## Topic 1: Conic Sections

### **Engaging Experience 1**

Title: Conic Sections Centered about the Origin with the white boards

**Suggested Length of Time:** 1-1/2 class periods (Parabolas-<sup>1</sup>/<sub>2</sub> class period, Circles/Ellipses-<sup>1</sup>/<sub>2</sub> class period, Hyperbolas-<sup>1</sup>/<sub>2</sub> class period)

Standards Addressed

Priority:

• Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

Supporting:

- Alg2.IF.A.2: Translate between equivalent forms of functions.
- ISTE-COMPUTATIONAL THINKER.5.A formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.
- AL.9-12.F-CS.1 Create graphs of conic sections, including parabolas, hyperbolas, ellipses, circles, and degenerate conics, from second-degree Equations.
- AL.9-12.F-CS.1.a Formulate equations of conic sections from their determining characteristics.

**Detailed Description/Instructions:** Students will be given equations of conic sections in standard form and will then sketch their graphs and identity their key features. Students will be given the graph of a conic and then write its equation in standard form. When given the key features of a conic, students will write the equation and graph. They can use technology to verify their answers.

Bloom's Levels: Understand Webb's DOK: 2

### **Engaging Experience 2**

Title: Transforming Conic Sections

Suggested Length of Time: 1-1/2 class periods (Parabolas-<sup>1</sup>/<sub>2</sub> class period, Circles/Ellipses-<sup>1</sup>/<sub>2</sub> class period, Hyperbolas-<sup>1</sup>/<sub>2</sub> class period). This experience will be done concurrently with engaging experience 1.

### **Standards Addressed**

Priority:

• Alg2.IF.A.1: Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.

Supporting:

- Alg2.IF.A.2: Translate between equivalent forms of functions.
- AL.9-12.F-CS.1 Create graphs of conic sections, including parabolas, hyperbolas, ellipses, circles, and degenerate conics, from second-degree Equations.
- AL.9-12.F-CS.1.a Formulate equations of conic sections from their determining characteristics.

**Detailed Description/Instructions:** Students will be given translated conic sections in general and standard form and tasked with graphing those sections. If the section is in general form they will need to convert it is standard form by completing the square.

Bloom's Levels: Analyze

Webb's DOK: 3

## Topic 2: Systems of Equations

**Engaging Experience 1** 

**Title:** Writing dependent systems in parametric form

Suggested Length of Time: 1/2 of a class period

### **Standards Addressed**

Priority:

• Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.

Supporting:

• Alg2.IF.A.2: Translate between equivalent forms of functions.

**Detailed Description/Instructions:** Students will solve linear systems and 3 variables that have infinite solutions. They will have to decided which type of dependent system they have been given and write the answer in the appropriate form, i.e. set-builder notation or parametric form. **Bloom's Levels:** Apply

Webb's DOK: 2

### **Engaging Experience 2**

Title: Extending Methods of solving Linear Systems to Nonlinear on White Boards Suggested Length of Time: 15 minutes

### **Standards Addressed**

Priority:

• Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.

**Detailed Description/Instructions:** The student will apply substitution, elimination, and graphing to systems involving nonlinear equations.

**Bloom's Levels:** Apply **Webb's DOK:** 2

### **Engaging Experience 3**

**Title:** Graphically solving systems of inequalities and applying to real world situations **Suggested Length of Time:** 1 class period

### Standards Addressed

Priority:

• Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.

**Detailed Description/Instructions:** Students will be presented with systems of inequalities involving linear and nonlinear inequalities and then use the process in linear programming. **Bloom's Levels:** Apply **Webb's DOK:** 2

## **Engaging Scenario**

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will do the linear programing extension at the end of chapter 5 in the textbook to find optimal solutions. Students will need to show the system of linear inequalities and the vertices of the feasible region.

# Summary of Engaging Learning Experiences for Topics

Торіс	Engaging Experience Title	Description	Suggested Length of Time
Conic Sections	Conic Sections Centered about the Origin with the white boards	Students will be given equations of conic sections in standard form and will then sketch their graphs and identity their key features. Students will be given the graph of a conic and then write its equation in standard form. When given the key features of a conic, students will write the equation and graph. They can use technology to verify their answers.	1-1/2 class periods (Parabolas- <sup>1</sup> / <sub>2</sub> class period, Circles/Ellipses- <sup>1</sup> / <sub>2</sub> class period, Hyperbolas- <sup>1</sup> / <sub>2</sub> class period)
Conic Sections	Transforming Conic Sections	Students will be given translated conic sections in general and standard form and tasked with graphing those sections. If the section is in general form they will need to convert it is standard form by completing the square.	1-1/2 class periods (Parabolas- <sup>1</sup> / <sub>2</sub> class period, Circles/Ellipses- <sup>1</sup> / <sub>2</sub> class period, Hyperbolas- <sup>1</sup> / <sub>2</sub> class period). This experience will be done concurrently with engaging experience 1
Systems of Equations	Writing dependent systems in parametric form	Students will solve linear systems and 3 variables that have infinite solutions. They will have to decided which type of dependent system they have been given and write the answer in the appropriate form, i.e. set-builder notation or parametric form.	<sup>1</sup> ∕2 of a class period
Systems of Equations	Extending Methods of solving Linear	The student will apply substitution, elimination, and graphing to systems involving nonlinear equations.	15 minutes

	Systems to Nonlinear on White Boards		
Systems of Equations	Graphically solving systems of inequalities and applying to real world situations	Students will be presented with systems of inequalities involving linear and nonlinear inequalities and then use the process in linear programming.	1 class period

## Unit 12: Systems and Matrices

Subject: College Algebra

Grade: 10, 11, 12

Name of Unit: Systems and Matrices

Length of Unit: 7 class periods

**Overview of Unit**: Students will learn basic matrix operations to evaluate expressions and solve real world applications. Students will use matrices to solve linear systems including augmented matrices, inverse matrices, and Cramer's Rule.

### **Priority Standards for unit:**

• Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.

### Supporting Standards for unit:

- N 406. Add two matrices that have whole number entries
- N 607. Use relations involving addition, subtraction, and scalar multiplication of vectors and of matrices
- N 706. Apply properties of matrices and properties of matrices as a number system
- ISTE-KNOWLEDGE COLLECTOR.3.D build knowledge by actively exploring realworld issues and problems, developing ideas and theories and pursuing answers and solutions.
- ISTE-COMPUTATIONAL THINKER.5.A formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.

Unwrapped Concepts (Students need to know)	Unwrapped Skills (Students need to be able to do)	Bloom's Taxonomy Levels	Webb's DOK
systems of equations that may include non-			
linear equations and inequalities.	Create	Create	3
systems of equations that may include non-			
linear equations and inequalities.	Solve	Apply	2

### **Essential Questions**:

- 1. How are matrices related to systems of linear equations?
- 2. Why are learning matrix solutions to systems beneficial?
- 3. How do I find and evaluate the determinant of a square matrix larger than a 2 x 2?

### **Enduring Understanding/Big Ideas**:

- 1. Linear systems can be rewritten as an augmented matrix or a matrix equation.
- 2. Technology can be used to solve linear systems extremely quickly when written as matrix.
- 3. By using expansion by minors, a square can be broken into several 2x2 matrices.

#### **Unit Vocabulary:**

Academic Cross-Curricular Words	Content/Domain Specific
	Augmented
	Inverse
	Row Echelon
	Gaussian
	Rows
	Columns
	Scalar
	Order (Dimensions)
	Element
	Determinant
	Minor
	Cofactor

**Resources for Vocabulary Development:** Textbook

## Topic 1: Matrices

**Engaging Experience 1** 

Title: Real-World Problem with Matrix Operations

Suggested Length of Time: 15 minutes

### **Standards Addressed**

Priority:

• Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.

Supporting:

- N 406. Add two matrices that have whole number entries
- N 607. Use relations involving addition, subtraction, and scalar multiplication of vectors and of matrices
- ISTE-KNOWLEDGE COLLECTOR.3.D build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.

**Detailed Description/Instructions:** Students will model real-world data using a system of equations and will perform the operations of addition, subtraction, and scalar multiplication. The website <u>https://meangreenmath.files.wordpress.com/2016/01/spacemath.png</u> contains a sample problem. Afterwards, students will model the same process using Microsoft Excel in order to make a connection between an abstract concept in mathematics and a concrete program that students use on a weekly basis.

**Bloom's Levels:** Create Webb's DOK: 3

### **Engaging Experience 2**

Title: Using Augmented Matrices to solve linear systems Suggested Length of Time: 1 class period Standards Addressed

Priority:

• Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.

Supporting:

- N 706. Apply properties of matrices and properties of matrices as a number system
- ISTE-COMPUTATIONAL THINKER.5.A formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.

### **Detailed Description/Instructions:**

Students will rewrite linear systems of equations as an augmented matrix and solve them using Gaussian and Gauss-Jordan Elimination. Students then will verify their answers by using a graphing calculator to put the matrix in Reduced Row Echelon Form. Bloom's Levels: Apply

Webb's DOK: 2

### **Engaging Experience 3**

Title: Using a Matrix Equation to solve linear systems Suggested Length of Time: 2 class periods Standards Addressed

Priority:

• Alg2.REI.B.1: Create and solve systems of equations that may include non-linear equations and inequalities.

### Supporting:

- N 706. Apply properties of matrices and properties of matrices as a number system
- ISTE-COMPUTATIONAL THINKER.5.A formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.

### **Detailed Description/Instructions:**

Students will rewrite linear systems of equations as a matrix equation and solve them using an Inverse Matrix and Cramer's Rule. Students will use a graphing utility to find the inverse and determinants of larger systems.

**Bloom's Levels:** Apply **Webb's DOK:** 2

### Engaging Scenario

**Engaging Scenario** (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will be handed a bag consisting of 4 types of coins and be told the total amount of money and number of coins. They students will also be given two pieces of information that will relate the amount of the different types of coins. Students will use this information to determine how many of each type of coin is in the bag. Students will need to produce the systems of equations and matrices but may use technology to solve. To confirm their results, the students will then open the bag and count the coins.

# Summary of Engaging Learning Experiences for Topics

Торіс	Engaging Experience Title	Description	Suggested Length of Time
Matrices	Real-World Problem with Matrix Operations	Students will model real-world data using a system of equations and will perform the operations of addition, subtraction, and scalar multiplication. The website <u>https://meangreenmath.files.wordpress.com/2016</u> /01/spacemath.png contains a sample problem. Afterwards, students will model the same process using Microsoft Excel in order to make a connection between an abstract concept in mathematics and a concrete program that students use on a weekly basis.	15 minutes
Matrices	Using Augmented Matrices to solve linear systems	Students will rewrite linear systems of equations as an augmented matrix and solve them using Gaussian and Gauss-Jordan Elimination. Students then will verify their answers by using a graphing calculator to put the matrix in Reduced Row Echelon Form.	1 class period
Matrices	Using a Matrix Equation to solve linear systems	Students will rewrite linear systems of equations as a matrix equation and solve them using an Inverse Matrix and Cramer's Rule. Students will use a graphing utility to find the inverse and determinants of larger systems.	2 class periods

## **Unit of Study Terminology**

<u>Appendices</u>: All Appendices and supporting material can be found in this course's shell course in the District's Learning Management System.

Assessment Leveling Guide: A tool to use when writing assessments in order to maintain the appropriate level of rigor that matches the standard.

**Big Ideas/Enduring Understandings:** Foundational understandings teachers want students to be able to discover and state in their own words by the end of the unit of study. These are answers to the essential questions.

**Engaging Experience:** Each topic is broken into a list of engaging experiences for students. These experiences are aligned to priority and supporting standards, thus stating what students should be able to do. An example of an engaging experience is provided in the description, but a teacher has the autonomy to substitute one of their own that aligns to the level of rigor stated in the standards.

**Engaging Scenario:** This is a culminating activity in which students are given a role, situation, challenge, audience, and a product or performance is specified. Each unit contains an example of an engaging scenario, but a teacher has the ability to substitute with the same intent in mind.

**Essential Questions:** Engaging, open-ended questions that teachers can use to engage students in the learning.

**<u>Priority Standards:</u>** What every student should know and be able to do. These were chosen because of their necessity for success in the next course, the state assessment, and life.

Supporting Standards: Additional standards that support the learning within the unit.

**Topic:** These are the main teaching points for the unit. Units can have anywhere from one topic to many, depending on the depth of the unit.

<u>Unit of Study:</u> Series of learning experiences/related assessments based on designated priority standards and related supporting standards.

<u>Unit Vocabulary:</u> Words students will encounter within the unit that are essential to understanding. Academic Cross-Curricular words (also called Tier 2 words) are those that can be found in multiple content areas, not just this one. Content/Domain Specific vocabulary words are those found specifically within the content.